Mechanisms of deep penetration of soft solids

Oliver A. Shergold and Norman A. Fleck*

Cambridge University Engineering Dept., Trumpington St., Cambridge, CB2 1PZ, UK

* Author for correspondence

Abstract

Micromechanical models are developed for the deep penetration of a soft solid by a flat-bottomed and by a sharp-tipped cylindrical punch. The soft solid is taken to represent mammalian skin and silicone rubbers, and is represented by an incompressible, hyper-elastic, isotropic solid described by a one term Ogden strain energy function. The flat-bottomed punch penetration model assumes that penetration is by the formation of a mode II ring crack that propagates ahead of the penetrator tip. The sharp-tipped punch penetration model assumes that penetration is by the formation of a planar mode I crack which is wedged open by the advancing punch.

In both models the steady-state penetration load is obtained by equating the work done in advancing the punch to the sum of the fracture work and the strain energy stored in the solid. For the case of a sharp penetrator, this calculation is performed by considering the opening of a plane strain crack by wedge loading, using a finite element approach. Analytical methods suffice for the flat-bottomed punch. In both models, the crack dimensions are determined by minimising the load on the punch with respect to crack geometry.

For both geometries of punch tip, the predicted penetration pressure increases with diminishing punch radius, and with increasing toughness and strain hardening capacity of solid. The penetration pressure for a flat-bottomed punch is two to three times greater than that for a sharp-tipped punch (assuming that the mode I and mode II toughnneses are equal), in agreement with experimental observations reported in a companion paper (Experimental Investigation Into the Penetration of Soft Solids, by O A Shergold and N A Fleck, submitted to the Journal of Biomechanics).

Keywords: deep penetration, injection, fracture mechanics, finite element method, soft solids

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1 INTRODUCTION

The deep penetration of soft solids by a punch is of widespread technological importance, with applications ranging from the piercing of mammalian skin by a hypodermic needle (or by a liquid jet) in administering an injection, to the failure of rubber seals or tires by the penetration of a foreign body.

Despite the ubiquitous nature of soft solid penetration, the existing literature provides little insight into the underlying mechanisms of penetration. The medical literature on skin injection, and the mechanical engineering literature on rubber penetration, indicate that deep penetration involves cracking of the soft solid, followed by substantial reversible deformation when the penetrator is removed [1-3]. A limited number of experimental studies reveal that the resultant crack geometry is sensitive to the punch tip geometry and material properties of the penetrated solid. Stevenson and Abmalek [3] showed that a flat-bottomed punch penetrates natural rubbers by the formation of a mode II ring crack that propagates ahead of the penetrator tip, as shown in Figure 1a. In a companion paper to the present study, Shergold and Fleck [4] observed that a sharp-tipped punch penetrates silicone rubber and skin by the formation of a planar mode I crack ahead of the tip; the crack faces are wedged open by the advancing punch, as shown in Figure 1b. They also confirmed that a flat-bottomed punch penetrates silicone rubber and skin by the mode II ring-crack mechanism of Stevenson and Abmalek [3]. The main purpose of the present paper is to develop quantitative models for the penetration of soft materials by flat-bottomed and sharp-tipped punches by the mechanisms shown in Figure 1. These models are justified by drawing upon salient experimental results taken from the companion paper [4].

The penetration mechanism observed for soft solids is different to that for strong solids such as metals, soils and polymers. Strong solids are penetrated by the formation and expansion of a spherical (or cylindrical) cavity at the penetrator tip, see Figure 1c [5-10]. Bishop et al. [5] modelled penetration by the expansion of a cavity in an elastic-ideally plastic solid, and argued that the penetration pressure is comparable to the cavitation pressure $p_c$, as defined by the pressure to expand the cavity from zero initial radius to a finite final radius. They showed that the cavitation pressure for a spherical cavity is close to that for a cylindrical cavity and so the precise details of the cavity shape are relatively unimportant. Typically for metals, $p_c$ is on the order of 4-5 times the uniaxial yield strength, depending upon the yield strain and the strain hardening rate. Wright et al. [10] have applied this model to the case of deep penetration of polymers, and they showed that the cavitation pressure $p_c$ becomes infinite when the strain hardening behaviour is strongly exponential in character. They argued that penetration involves a cracking mechanism in addition to hole expansion for polymers displaying exponential hardening, and they provided experimental evidence by performing a series of tests on transparent polycarbonate blocks. Wright et al. were able to make accurate predictions by including a ‘hackle zone’ within the cavity expansion model.

The structure of the paper is as follows. First, a constitutive description is given for the stress versus strain response of rubber-like materials with a wide range of strain hardening behaviours. This constitutive description is used in both the flat-bottomed
and sharp-tipped penetration models. Then, penetration models are presented for the penetration of an incompressible, hyperelastic, isotropic solid by a flat-bottomed punch and a sharp-tipped punch. For each model the crack geometry is obtained by minimising the load for steady state penetration. The flat-bottomed punch model is solved analytically, whilst a finite element approach is required for the sharp-tipped punch penetration model. For each model the steady-state penetration load is calculated for a range of material responses (strain hardening and toughness) and punch diameters. Direct comparisons of the predictions are made with measured penetration loads (and crack dimensions) taken from the companion experimental study [4].

Figure 1: Penetration mechanisms of a soft solid by (a) a flat-bottomed punch and (b) a sharp-tipped punch, and (c) penetration of a strong solid by a flat-bottomed or sharp-tipped punch.
2 RUBBER-LIKE CONSTITUTIVE MODELS

The constitutive description of the soft solid is taken to be the Ogden [11] model for an incompressible, isotropic, hyper-elastic solid. This model describes a wide range of strain hardening behaviours and is readily available within finite element codes such as ABAQUS. Throughout this paper we use a modified form of the one-term Ogden strain energy density function

$$\phi = \frac{2\mu}{\alpha^2} \left[ \lambda_1^\alpha + \lambda_2^\alpha + \lambda_3^\alpha - 3 \right]$$  \hspace{1cm} (1)

where $\phi$ is the strain energy density per undeformed unit volume, $\alpha$ is a strain hardening exponent and $\mu$ has the interpretation of the shear modulus under infinitesimal straining. This form of the Ogden strain energy density is consistent with that used by the finite element analysis software ABAQUS [12]. Table 1 gives typical values of the Ogden constants for three grades of silicone rubber and human skin, as determined by Shergold and Fleck [4]. Values for the toughness $J_{IC}$ are included in Table 1; the values for rubbers are taken from Shergold and Fleck [4] while the toughness value of human skin has been measured by Pereira et al. [13].

<table>
<thead>
<tr>
<th>Solid</th>
<th>Grade</th>
<th>$\mu$ (MPa)</th>
<th>$\alpha$</th>
<th>$J_{IC}$ (kJ m$^{-2}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silicone rubber</td>
<td>Sil8000</td>
<td>2.7</td>
<td>2.5</td>
<td>9.1</td>
</tr>
<tr>
<td>Silicone rubber</td>
<td>B452</td>
<td>0.4</td>
<td>3.0</td>
<td>7.9</td>
</tr>
<tr>
<td>Human skin</td>
<td></td>
<td>0.11</td>
<td>9.0</td>
<td>2.5</td>
</tr>
</tbody>
</table>

Table 1: Curve fits of the Ogden constitutive law (1), and toughness values of solids used in penetration experiments reported by Shergold and Fleck [4].

3 FLAT-BOTTOMED PUNCH PENETRATION MODEL

We begin by summarising the model of Stevenson and Abmalek [3] for the penetration of a soft solid by a flat-bottomed circular cylindrical punch. The development given below makes use of the Ogden formulation of elastic strain energy, where Stevenson and Abmalek [3] made use of the Mooney-Rivlin formulation. (The stresses versus strain curves given in our companion study reveal high strain hardening behaviour at large strains, which the Ogden description can capture, but the Mooney-Rivlin description cannot).

Consider a frictionless, flat-bottomed, rigid cylindrical punch of radius, $R$, penetrating a semi-infinite block, as sketched in Figure 2a. A ring crack propagates ahead of the penetrator tip forming a column, which in the undeformed configuration has a radius $b$ and a length $\ell$, see Figure 2b. The ring defines a hole within the solid with radius $b$ and length $\ell$. The punch shortens the column from an undeformed length $\ell$ to a compressed length $h$. Simultaneously the radius of the column increases from $b$ to $R$, due to incompressibility. The penetration load $P_I$ is estimated by equating the work done by advance of the punch to the sum of the crack work and the strain energy $S$ in the penetrated solid. There are two contributions to $S$: that due to compression of the column, $S_C$, and that due to expansion of the hole, $S_H$. 
Consider steady state penetration of the punch at a fixed load $P_F$. In the current state the punch has descended by a depth $h$. The work done by the punch upon advancing by an increment $\delta h$ is given by

$$P_F \cdot \delta h = 2\pi \cdot b \cdot J_{IIc} \cdot \delta \ell + \frac{\partial S_C}{\partial \ell} \cdot \delta \ell + \frac{\partial S_H}{\partial \ell} \cdot \delta \ell$$

(2)

where $J_{IIc}$ is the toughness of the material to mode II crack propagation. The quantity $\frac{\partial S_C}{\partial \ell}$ is the work done per unit depth (in the undeformed configuration) in order to compress the column beneath the punch by an axial stretch factor of $\lambda_z = \frac{b^2}{R^2}$, upon invoking incompressibility. $\frac{\partial S_H}{\partial \ell}$ is the energy stored within unit thickness (undeformed) of the surrounding solid, external to the hole. Explicit expressions for $\frac{\partial S_C}{\partial \ell}$ and $\frac{\partial S_H}{\partial \ell}$ are obtained below. It is convenient at this point to use dimensional analysis to write

$$\frac{\partial S_C}{\partial \ell} + \frac{\partial S_H}{\partial \ell} = \pi \mu R^2 f\left(\frac{b}{R}\right)$$

(3)

where the non-dimensional function $f\left(\frac{b}{R}\right)$ remains to be determined.

Substitution of (3) into (2) defines the average penetration pressure $p_F$ on the end of the punch as

$$\frac{p_F}{\mu} = \frac{P_F}{\pi R^2 \mu} = \left[\frac{2b J_{IIc}}{R \mu R} + f\left(\frac{b}{R}\right)\right] \frac{\partial \ell}{\partial h}$$

(4)
Since the solid is taken as incompressible, volume conservation of the column gives

\[ \pi R^2 (\ell - h) = \pi b^2 \ell \]  

(5)

and (4) can be re-written as

\[ \frac{p_e}{\mu} = \left[ 1 - \left( \frac{b}{R} \right)^2 \right]^{-1} \left[ \frac{2b J_{IC}}{R} \mu R + f \left( \frac{b}{R} \right) \right] \]  

(6)

### 3.1 Calculation of the stored strain energy

We now define expressions for \( \frac{\partial S_c}{\partial \ell} \) and \( \frac{\partial S_h}{\partial \ell} \). First consider the strain energy stored in the solid due to expansion of the hole.

#### 3.1.1 Strain energy stored due to hole expansion

Stevenson and Abmalek [3] follow the approach of Bishop et al. in determining \( \frac{\partial S_h}{\partial \ell} \). However, for materials where the stress versus strain response is defined by a strain energy function, we propose a simpler method to determine \( \frac{\partial S_h}{\partial \ell} \). Consider a representative cylindrical annulus of the solid in the undeformed configuration. The annulus has an initial radius \( s \), thickness \( ds \) and unit height, see Figure 3a, and expands to a radius \( r \), thickness \( dr \) and unit height, as shown in Figure 3b.

![Figure 3: Expansion of a cylindrical annulus of the solid from a radius \( s \) to a radius \( r \), on expansion of the hole from a radius \( b \) to a radius \( R \), (a) undeformed configuration, (b) deformed configuration.](image)

Volume conservation for the incompressible solid dictates that

\[ r^2 - R^2 = s^2 - b^2 \]  

(7)
and \( \lambda_r \lambda_\theta \lambda_z = 1 \), where \( \lambda_r \), \( \lambda_\theta \) and \( \lambda_z \) are the principal stretch ratios in a polar cylindrical co-ordinate system.

The assumption of plane strain hole expansion implies \( \lambda_z = 1 \) and hence

\[
\lambda_\theta = \frac{1}{\lambda_r} = \frac{r}{s}
\]

The Ogden expression (1) for the strain energy density \( \phi(s) \) of the annulus in the undeformed configuration is

\[
\phi(s) = \frac{2\mu}{\alpha} \left[ \left( \frac{r}{s} \right)^\alpha + \left( \frac{s}{r} \right)^\alpha - 2 \right]
\]

Hence the stored strain energy within the solid, associated with hole expansion from a radius \( b \) to a radius \( R \), is

\[
\frac{\partial S_H}{\partial \ell} = 2\pi s \phi(s) ds = \int_b^R 2\pi r \phi(r) dr 
\]

Upon making the substitution

\[
\eta = \left( \frac{r}{R} \right)^2
\]

(9) and (10) give

\[
\frac{\partial S_H}{\partial \ell} = \pi R^2 \int_1^\infty \frac{2\mu}{\alpha} g \left( \eta, \frac{b}{R} \right) d\eta
\]

where

\[
g \left( \eta, \frac{b}{R} \right) = \left[ \left( 1 - \left( \frac{b}{R} \right)^2 \right)^\alpha \right] + \left[ \left( 1 - \left( \frac{b}{R} \right)^2 \right)^\alpha \right] - 2
\]

The integral (12) can be determined analytically for the choice \( \alpha = 2 \), and \( \frac{\partial S_H}{\partial \ell} \) is then given by

\[
\frac{\partial S_H}{\partial \ell} = \pi \mu R^2 \left( \frac{b^2}{R^2} - 1 \right) \ln \left( \frac{b}{R} \right)
\]
3.1.2 Stored strain energy due to compression of the column

Now consider the strain energy \( \frac{\partial S_C}{\partial \ell} \) associated with compression of a column of unit undeformed length and radius \( b \) to a current radius of \( R \). For a polar cylindrical co-ordinate system the principal stretch ratios for column compression are

\[
\lambda_r = \lambda_\theta = \frac{1}{\sqrt{\lambda_z}} = \frac{R}{b}
\]

Hence the one term Ogden strain energy function is

\[
\phi = \frac{2\mu}{\alpha^2}(2\lambda_r^\alpha + \lambda_z^{2\alpha} - 3)
\]

and the strain energy in the column \( S_C \) of radius \( b \) and unit depth is

\[
\frac{\partial S_C}{\partial \ell} = \pi R^2 \frac{2\mu}{\alpha^2} \left[ 2 \left( \frac{b}{R} \right)^{2-\alpha} + \left( \frac{b}{R} \right)^{2\alpha+2} - 3 \left( \frac{b}{R} \right)^2 \right]
\]

3.1.3 Non-dimensional stored strain energy function

Recall that the sum \( \frac{\partial S_C}{\partial \ell} + \frac{\partial S_H}{\partial \ell} \) was expressed in terms of a non-dimensional function \( f \left( \frac{b}{R} \right) \). Expressions (12) and (17) allow \( f \left( \frac{b}{R} \right) \) to be stated explicitly as

\[
f \left( \frac{b}{R} \right) = \frac{2}{\alpha^2} \left[ 2 \left( \frac{b}{R} \right)^{2-\alpha} + \left( \frac{b}{R} \right)^{2\alpha+2} - 3 \left( \frac{b}{R} \right)^2 \right] + \int_1^\infty g \left( \eta, \frac{b}{R} \right) \, d\eta
\]

where \( g \left( \eta, \frac{b}{R} \right) \) is given by (13).

3.2 Prediction of the diameter of the cylindrical crack

We note from (18) that \( p_F \) depends upon an assumed value of \( b/R \), which we shall now determine. Consider how \( p_F/\mu \), as given in (6), varies with \( b/R \). Contributions to \( p_F/\mu \) are made by the mechanisms of crack formation, hole expansion and column compression. For low values of \( b/R \) the stored strain energy associated with hole expansion is the dominant contributor to \( p_F/\mu \), whilst at high values of \( b/R \) the work of crack formation is the dominant contributor. These separate contributions are shown in Figure 4 for a solid with \( \alpha \) taken to be 2 for simplicity. A global minimum in \( p_F/\mu \) exists at an intermediate value of \( b/R \).
It is argued that stable penetration occurs at this minimum value of pressure. The relation (6) achieves a minimum value \( p_{F_{\text{min}}} \) when \( b/R \) satisfies the relation

\[
\frac{J_{\text{IIc}}}{\mu R} \left[ 1 + \left( \frac{b}{R} \right)^2 \right] = \frac{1}{2} \left[ 1 - \left( \frac{b}{R} \right)^2 \right] f' \left( \frac{b}{R} \right) \left( \frac{b}{R} \right) f \left( \frac{b}{R} \right)
\]

where \( f(b/R) \) is

\[
f' \left( \frac{b}{R} \right) = \frac{2}{\alpha^2} \left\{ 2(2-\alpha) \left( \frac{b}{R} \right)^{1-\alpha} + 2(\alpha+1) \left( \frac{b}{R} \right)^{\alpha+1} - 6 \left( \frac{b}{R} \right) \right\} + \frac{d}{d\left( b/R \right)} \left[ \int_{R}^{\infty} g \left( \frac{b}{R}, \eta \right) d\eta \right]
\]

Note that this value for \( b/R \) only depends upon the non-dimensional groups \( J_{\text{IIc}}/\mu R \) and \( \alpha \). Consequently the minimum value \( p_{F_{\text{min}}}/\mu \) is a function of \( J_{\text{IIc}}/\mu R \) and \( \alpha \). The relations (6) and (19) have been evaluated numerically and the predicted dependence of \( p_{F_{\text{min}}}/\mu \) upon \( J_{\text{IIc}}/\mu R \), with corresponding values of \( b/R \), are shown in Figure 5. As expected \( p_{F_{\text{min}}}/\mu \) increases and \( b/R \) decreases with increasing \( J_{\text{IIc}}/\mu R \).
Figure 5: Prediction of (a) $\frac{p_{F_{\text{min}}}}{\mu}$ versus $J_{\text{IIc}}/\mu R$, and (b) $b/R$ versus $J_{\text{IIc}}/\mu R$, for the penetration of a soft solid by a flat punch.
4 SHARP-TIPPED PUNCH PENETRATION MODEL

We now consider the penetration of a soft solid by a sharp-tipped punch. We calculate the steady-state penetration force to advance and open a crack at the punch tip. The model includes a condition for determining the stable crack length along the flank of the punch.

Consider a frictionless, rigid cylindrical punch of radius, \( R \), with a conical tip pushed into a semi-infinite block, as shown in Figure 6a. The solid tears and opens at the tip of the punch. The detailed solution for the punch tip requires a full 3D calculation; however we can consider punch advance by \( \delta \ell \) as equivalent to creating a plane strain crack of length \( 2a \) in a slice of thickness \( \delta \ell \) and then opening the crack to accommodate the punch. This energy approach is accurate when the strain energy density in each material element is independent of strain path.

Consider the steady state advance of the punch by an axial increment \( \delta \ell \) due to a load \( P_S \). The work done by the punch in effecting this advance is \( P_S \cdot \delta \ell \). This work increment balances the energy \( \delta W_C \) required to form a crack of length \( 2a \) in a solid slice of thickness \( \delta \ell \), see Figure 6b, and the strain energy stored in the solid \( \delta S_E \) on opening the crack to accommodate a circular cylindrical inclusion of radius \( R \), see Figure 6c. Hence,

\[
P_S \cdot \delta \ell = \delta W_C + \delta S_E
\]  

Figure 6: (a) penetration of a soft solid by a sharp-tipped punch, (b) formation of a plane strain crack of length \( 2a \) in an infinite slice of thickness \( \delta \ell \), (c) opening of the crack by an expanding circular cylinder to a final radius \( R \).

The work required to create the crack, \( \delta W_C \), is determined by the mode I toughness of the material, \( J_{IC} \) and is given by

\[
\delta W_C = 2J_{IC} \cdot a \cdot \delta \ell
\]

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In order to calculate the work $S_E$ required to wedge open the crack, we consider the auxiliary problem of expanding a circular wedge of radius $R'$ from $R' = 0$ to a final value of $R' = R$. It is convenient to write $\delta S_E$ as

$$\delta S_E = R^2 \cdot \mu \cdot \delta \ell \cdot h \left( \frac{a}{R} \right)$$

(23)

where the dimensionless function $h \left( \frac{a}{R} \right)$ is evaluated explicitly using a finite element procedure outlined below.

The sum of (22) and (23) gives the work done by the punch,

$$\frac{P_s}{\mu} \cdot \delta \ell = 2J_{\infty} \cdot a \cdot \delta \ell + R^2 \cdot \mu \cdot \delta \ell \cdot h \left( \frac{a}{R} \right)$$

(24)

and the average penetration pressure $p_S$ on the punch follows immediately as

$$\frac{p}{\mu} = \frac{P}{\pi R^2 \mu} = \frac{2}{\pi} \left( \frac{J_{\infty}}{\mu R} \right) \left( \frac{a}{R} \right) + \frac{1}{\pi} h \left( \frac{a}{R} \right)$$

(25)

4.1 Prediction of the crack length in steady state penetration

Figure 7 shows how $p_S/\mu$, as given in (25), varies with $a/R$ for the choice $\alpha = 3$. Also shown are the separate contributions to $p_S/\mu$ from the work of crack formation and the stored strain energy associated with crack opening. For low values of $a/R$ the strain energy associated with crack opening is the dominant contributor to $p_S/\mu$, whilst at high values of $a/R$ the work of crack formation is the dominant contributor. The pressure attains a global minimum value $p_{Smin}$ at an intermediate value of $a/R$. 
It is argued that stable penetration occurs at this minimum value of pressure. The relation (25) for $p_S/\mu$ achieves a minimum value when $a/R$ satisfies the relation

$$2 \frac{J_{IC}}{\mu R} + h'(\frac{a}{R}) = 0$$  \hspace{1cm} (26)

This expression is used to obtain the value of the length of crack formed during steady state penetration. We note that at the tip of the open crack along the flanks, the plane strain energy release rate $J_{PS}$ for an advance in crack length in a slice of thickness $\delta \ell$, may be expressed as

$$J_{PS} = -\frac{1}{2} \frac{\partial S_E}{\partial a} = -\frac{1}{2} \mu R h''(\frac{a}{R})$$  \hspace{1cm} (27)

Substitution of (26) into (27) gives the relation between the energy release rate at the tip of the plane strain crack and the material toughness [14-16]

$$J_{PS} = J_{IC}$$  \hspace{1cm} (28)

We note in passing that the crack along the flanks of the punch is stable: the plot of $J_{PS}$ versus crack length $a$ reveals that

$$\frac{\partial J_{PS}}{\partial a} < 0$$  \hspace{1cm} (29)

at fixed $p_{S\text{min}}$. 

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Figure 7: Contributions to $p_S/\mu$ from crack toughness and strain energy due to crack opening ($\alpha=3, J_{IC}/\mu R=0.23$)
4.2 A finite element approach to evaluate \( h(a/R) \) and \( h'(a/R) \)

A finite element approach is used to determine \( h(a/R) \) and \( h'(a/R) \) for selected values of \( \alpha \). The finite element procedure is shown in Figure 8. Considerations of symmetry allow us to model only a quadrant of the crack and punch. The initial configuration is taken to be a closed crack of length \( 2a \) within a square block of side length \( 40a \). A rigid frictionless punch of radius \( R \) is in point contact with the midpoint of the crack. Plane strain, 8 noded, quadrilateral elements are used to mesh the hyperelastic solid block. The region around the crack tip is meshed using a radial configuration of elements centred at the crack tip, as shown in Figure 9. Triangular elements are required for the innermost circumference of elements at the crack tip. These triangular elements are formed by collapsing one side of a four-sided element to a point [17]. The crack tip is thereby allowed to blunt under the applied loading. The remainder of the block is meshed using a rectangular grid of elements. Appropriate roller-boundary conditions are prescribed on symmetry planes as sketched in Figure 8, with the crack flanks traction-free where not in contact with the frictionless punch. The punch is displaced in incremental steps up to an indentation depth of magnitude \( R \). The stored strain energy and plane strain energy release rate at the crack tip are determined in the final state.

![Figure 8: Sketch of the finite element model for a plane strain crack wedged open by a circular cylindrical punch.](image)

![Figure 9: Mesh detail of the radial elements used to model the region around the crack tip.](image)
4.3 Results of the finite element analysis

The dependence of $\frac{p_{S\text{min}}}{\mu}$, and corresponding values of $a/R$, have been evaluated using the finite element approach. The results of this analysis are shown in Figure 10. As expected $\frac{p_{S\text{min}}}{\mu}$ increases and $a/R$ diminishes with increasing $J_{IC}/\mu R$.

Figure 10: Prediction of (a) $\frac{p_{S\text{min}}}{\mu}$ versus $J_{IC}/\mu R$, and (b) $\frac{a}{R}$ versus $J_{IC}/\mu R$, for the penetration of a soft solid by a sharp punch.
5 COMPARISON OF THE MODEL PREDICTIONS WITH MEASURED PENETRATION PRESSURES

A comparison of the predicted penetration pressures $p_{F_{\text{min}}}/\mu$ and $P_{S_{\text{min}}}/\mu$ versus $J/\mu R$ is plotted in Figure 11. The penetration pressure for a flat-bottomed punch $p_{F_{\text{min}}}/\mu$ is two to three times greater than that for a sharp-tipped punch, $p_{S_{\text{min}}}/\mu$, assuming that $J_{\text{IC}}$ is taken to equal $J_{\text{IC}}$ (reasonable for a rubber-like material).

The penetration model for a sharp-tipped punch is now compared with measured penetration pressures as reported by Shergold and Fleck [4] on two silicone rubbers (B452 and Sil8000) and on human skin. Additionally, the model is compared with the penetration measurements of Frick et al. [18] on sheepskin. They performed tests in vitro using $\Omega1.0$ mm suture needles, and here we assume that the stress versus strain response and toughness of sheep skin is adequately represented by the measured data for human skin. Figure 12a shows the predicted response of $p_{S_{\text{min}}}/\mu$ versus $J_{\text{IC}}/\mu R$ for the sharp-tipped punch penetration model, whilst the relation between the $a/R$ and $J_{\text{IC}}/\mu R$ is shown in Figure 12b. The results of the penetration experiments are included. In the experimental investigations the independent parameter $J_{\text{IC}}/\mu R$ was varied by selecting a range of punch radii $R$. There is reasonable agreement between the prediction and the measurements, although the crack lengths observed in Sil8000 are approximately twice as long as those predicted by the model.

The flat-bottomed punch penetration model is compared with the experiments of Shergold and Fleck [4] on silicon rubbers (B452 and Sil8000) and human skin, see Figure 13a for $p_{F_{\text{min}}}/\mu$ versus $J_{\text{IC}}/\mu R$ and Figure 13b for $b/R$ versus $J_{\text{IC}}/\mu R$. The predicted penetration pressures are adequate for the rubbers and skin. Although, the predicted hole diameter is satisfactory for the rubbers, the measured value for skin is about twice the measured diameter. A possible explanation lies in the fact that the hole diameter for the skin is a surface measurement, and the measurements on rubber reveal that the hole diameter at the surface is about twice the value at depth.
Figure 11: Comparison of $p/\mu$ versus $J/\mu R$ for penetration of a solid by a flat-bottomed punch and a sharp-tipped punch.
Sil8000 ($\alpha=2.5$, $J_{IC}=9.1$ kJ m$^{-2}$) Human skin ($\alpha=9.0$, $J_{IC}=2.5$ kJ m$^{-2}$) B452 ($\alpha=3.0$, $J_{IC}=7.9$ kJ m$^{-2}$) Sheep skin ($\alpha=9.0$, $J_{IC}=2.5$ kJ m$^{-2}$)

Figure 12: (a) $p/\mu$ versus $J_{IC}/\mu R$ and (b) $a/R$ versus $J_{IC}/\mu R$, for penetration of a soft solid by a sharp punch.
Sil8000 ($\alpha=2.5$, $J_{IIc}=9.1$ kJ m$^{-2}$)  
Human skin ($\alpha=9.0$, $J_{IIc}=20$ kJ m$^{-2}$)  
B452 ($\alpha=3.0$, $J_{IIc}=7.9$ kJ m$^{-2}$)

Figure 13: (a) $p/\mu$ versus $J_{IIc}/\mu R$ and (b) $p/\mu$ versus $J_{IIc}/\mu R$, for penetration of a soft solid by a flat punch.
6 CONCLUDING REMARKS

The two penetration models introduced above give reasonable predictions for the pressure required to penetrate a soft solid with either a sharp-tipped or flat-bottomed punch. The penetration pressure increases with diminishing radius of punch, and with increasing toughness and strain hardening capacity. Previous penetration models, based upon the expansion of a cavity, without cracking, are unable to predict realistic penetration pressures.

The use of a sharpened hypodermic needle to administer a injection is a long established technique. The current study reveals that such an injection leads to the formation of a crack rather than a residual hole in the dermis. Consequently, the wound can self-seal, and heal quickly, with minimal risk of infection.

We have assumed in our analysis that skin is transversely isotropic. In reality, the dermis is oriented along the Langer’s lines, and is more accurately represented as an orthotropic solid. It is possible to extend the above models to the orthotroipic case, and this warrants future investigation.

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8 REFERENCES


