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FINITE ELEMENT ANALYSIS OF PLASTICITY-INDUCED CRACK CLOSURE UNDER PLANE STRAIN CONDITIONS

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Abstract—In this study, fatigue crack growth in an elastic–perfectly plastic solid is modelled using a two-dimensional finite element analysis. Plane deformations are enforced and crack advance is simulated by node release. The analysis suggests that crack closure occurs for a transient period of crack growth, with a residual wedge of material being left on the crack flanks near the initial location of the crack tip. The influence of specimen geometry and load level upon closure response is determined, and explained in terms of the "T-stress". This stress is the non-singular constant second term in the series expansion of the normal stress parallel to the crack plane. It was found that the crack closure behaviour is insensitive to a change of the material properties σ_y/E and ν, where σ_y is the yield stress, E is Young's modulus and ν is Poisson's ratio.

INTRODUCTION

THERE IS no general agreement over the occurrence of plasticity-induced crack closure under plane strain conditions. The following argument has been used to suggest that plasticity-induced closure cannot occur when plane deformations are enforced. Assume that crack closure is associated with the creation of extra volume of material on the crack flanks. Since plastic deformation occurs with no dilatation, there is no mechanism for creating this extra volume of material when plane strain conditions prevail; thus crack closure cannot occur.

The error in this argument is displayed by a simple counter-example. Consider a semi-infinite crack in an infinite block of incompressible material, such as rubber. When remote loading is applied the crack opens, and the combined volume of the specimen and open crack increases. Apparently, material has been lost along the crack flanks in order to provide for opening of the crack. In reality, material remote from the crack tip has been displaced. In like manner, plasticity-induced crack closure may occur by material displacements *remote* from the crack tip.

Alternatively, it may be imagined that plasticity-induced crack closure occurs by the *local redistribution of material* near the crack tip in an elastic–plastic solid. Fleck and Newman [1] have shown that a growing fatigue crack has a similar crack opening profile at the maximum stress intensity of the fatigue cycle, $K_{\max}$, to that of a tearing crack under constant $K = K_{\max}$. The fatigue crack unloads, however, like a stationary crack. Since the crack opening displacement near the crack tip is greater for a stationary crack than for a tearing crack it follows that a growing fatigue crack can shut prematurely under tensile loads [1].

Larsson and Carlsson [2] and Rice [3] have shown that the influence of specimen geometry upon the small scale yielding response of stationary cracks may be accounted for in terms of the "T-stress". An asymptotic expansion for the in-plane stresses in an elastic solid yields

$$\begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{bmatrix} = \frac{K}{\sqrt{r}} \begin{bmatrix} f_{xx}(\theta) & f_{xy}(\theta) \\ f_{yx}(\theta) & f_{yy}(\theta) \end{bmatrix} + \begin{bmatrix} T & 0 \\ 0 & 0 \end{bmatrix} + \begin{array}{l} \text{terms which} \\ \text{vanish at} \\ \text{crack tip.} \end{array} \quad (1)$$

Here (x, y) is the plane of straining, the crack coincides with the x-axis, the crack tip is at the origin and $r = \sqrt{x^2 + y^2}$, $\theta = \tan^{-1}(y/x)$ are polar co-ordinates centred at the origin. The universal functions $f_{ij}(\theta)$ relate to the "K-field", and are given by Westergaard [4]. The portion of the non-singular stress field which does not vanish at the crack tip amounts to a uniform stress $\sigma_{xx} = T$ acting parallel to the crack plane. By first determining T in terms of the applied load for each of their specimens, Larsson and Carlsson showed that the crack opening displacement, δ, and plastic zone distribution for a stationary crack in an elastic–perfectly plastic solid are unique functions of K and T, for a range of geometries.

Fleck and Newman [1] have suggested that the effect of specimen geometry upon the solution for a tearing crack and a growing fatigue crack may be rationalised in terms of the T-stress.

Consider a growing fatigue crack, under constant K_{max} and $R(= K_{min}/K_{max})$. The ratio of the crack opening load P_{op}, to the maximum load of the fatigue cycle P_{max}, is given by

$$\frac{P_{op}}{P_{max}} = f\left(R, \frac{T_{max}}{\sigma_y}, \frac{\Delta a}{K_{max}^2/\sigma_y^2}, \frac{\sigma_y}{E}, \nu \right) \tag{2}$$

where f is an unknown function, and T_{max}/σ_y is the T-stress corresponding to K_{max}, divided by the yield stress for an elastic–perfectly plastic solid. The amount of crack advance is given by Δa, Young's modulus by E and Poisson's ratio by ν.

The purpose of the present theoretical study is to examine the influence of T_{max}/σ_y, $\Delta a/(K_{max}^2/\sigma_y^2)$, σ_y/E and ν on the crack closure response of a growing fatigue crack, at a load ratio $R = 0$. The previous study by Fleck and Newman [1] examined the influence of R upon the crack closure behaviour. They found that crack closure occurs only for $R < 0.3$.

2. THE FINITE ELEMENT MODEL

The finite element program developed by Newman [5] was employed; Newman has used this program to study crack closure under plane stress conditions [5].

In the present study, it was assumed that the material behaves in an elastic–perfectly plastic manner, with yield defined by the von Mises criterion. Unless otherwise stated, $\sigma_y/E = 1/200$ and $\nu = 0.3$, which are typical properties of high strength aluminium alloys. The finite element mesh consisted of 1967 two dimensional constant strain triangular elements with 2114 degrees of freedom. In order to avoid "plane strain locking" of the elements, the elements near the crack tip were arranged to form arrays of squares and their diagonals [6] (Fig. 1a).

Two types of boundary condition were employed in order to simulate a centre cracked panel (CCP) and a bend specimen (Fig. 1b). Fictitious springs were used to change the boundary conditions associated with crack growth, crack closure or crack opening. For free nodes along the crack surfaces, the spring stiffness was set to zero and for fixed nodes the stiffness was assigned extremely large values, see Newman [5] for details. When plotting plastic zone distributions it was assumed that an element had yielded when the effective stress in the element was greater than 0.98 σ_y. A more stringent criterion led to the frequent occurrence of an elastic element being completely surrounded by yielded elements, which is physically unreasonable. The size of the smallest elements near the crack tip was 0.00078125 w, where w is the width of the specimens, defined in Fig. 1b.

Fatigue crack growth was simulated by release of the crack tip node at K_{max}, followed by a single loading cycle $K_{max} \rightarrow K_{min} \rightarrow K_{max}$, Fig. 2. This process was then repeated. The applied loads were shed with increasing crack length in order to ensure that the specimen suffered a constant $\Delta K = K_{max} - K_{min}$, and a load ratio, R, of zero. Typically, a crack was advanced by 25 small element sizes from an initial crack length, a_0, of 0.486719 w to a final crack length, a_f, of 0.50625 w.

2.1. *Elastic analysis*

A preliminary elastic analysis was used to determine the T-stress for both specimen geometries, at $a/w = 0.5$. The T-stress was determined by extrapolating the stress σ_{xx} in elements behind the crack tip to $r = 0$, as outlined by Larsson and Carlsson [2]. It was found that $T\sqrt{w}/K = -0.786$ ($T/\sigma_{nom} = -1.14$) for the CCP geometry, while $T\sqrt{w}/K = 0.160$ ($T/\sigma_{nom} = 0.294$) for the bend specimen. The nominal stress, σ_{nom}, is defined in Fig. 1b for both geometries. It was assumed that $T\sqrt{w}/K$ showed little variation with crack length, over the small range in crack lengths considered in this study.

3. EFFECT OF LOAD LEVEL AND SPECIMEN GEOMETRY ON CLOSURE RESPONSE

The ratio P_{op}/P_{max} is given in Fig. 3 as a function of $\Delta a/(K_{max}^2/\sigma_y^2)$ for the bend and CCP geometries, and K_{max} equal to 0.22 $\sigma_y \sqrt{w}$ and to 0.44 $\sigma_y \sqrt{w}$. A K level of 0.44 $\sigma_y \sqrt{w}$ is just below

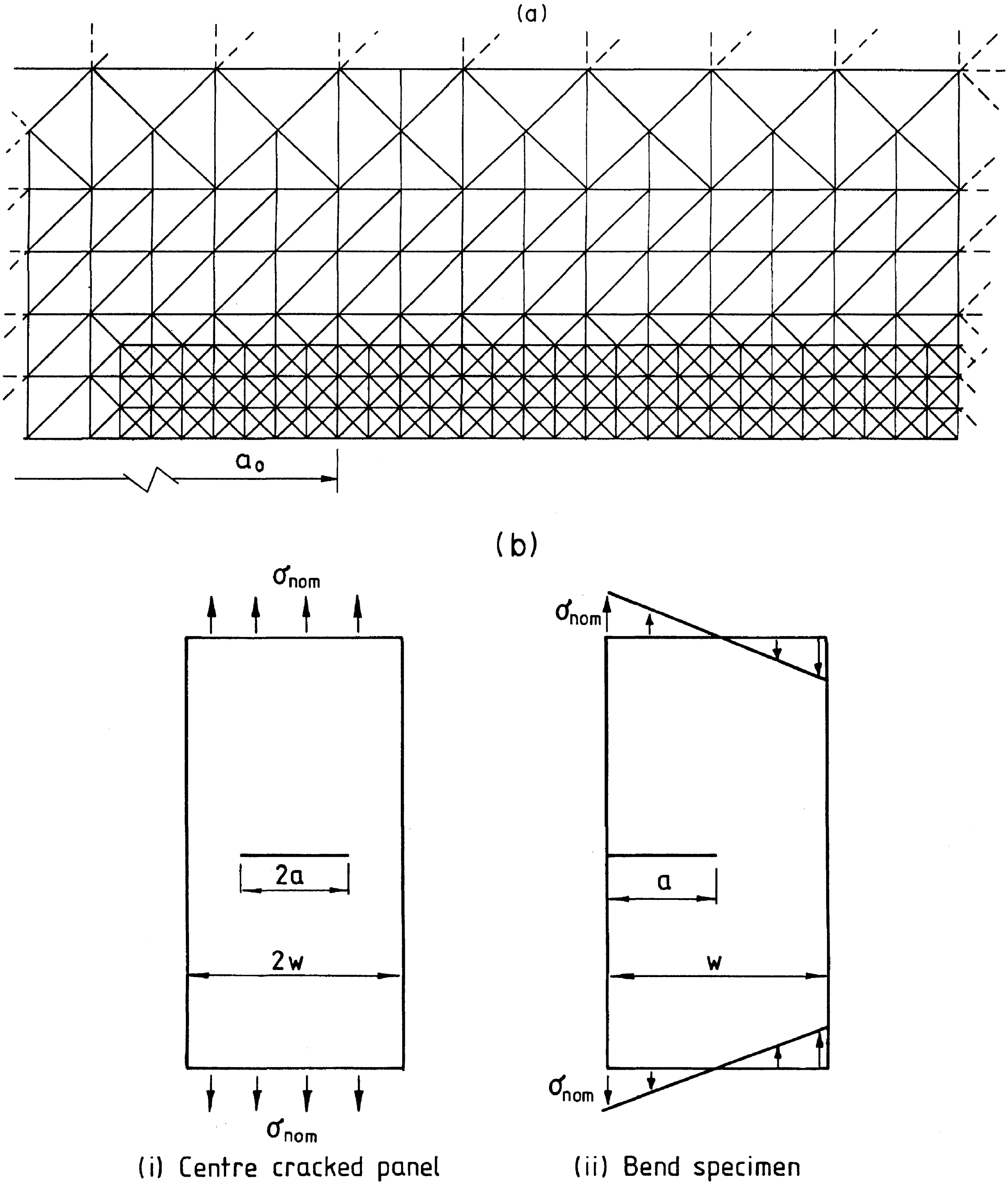

Fig. 1. (a) Mesh near crack tip. (b) Specimen geometries.

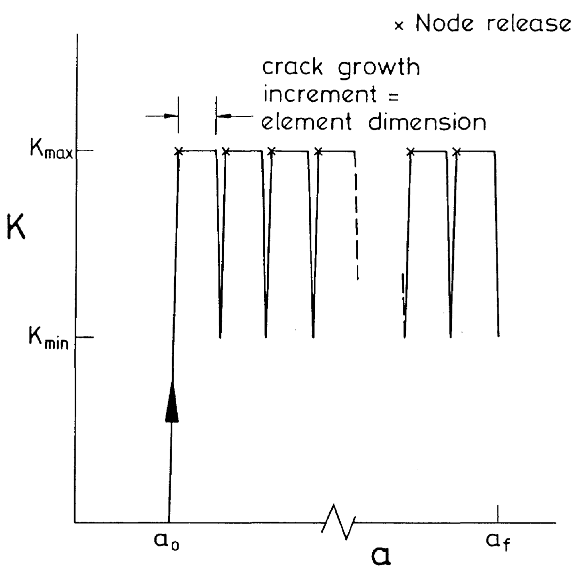

Fig. 2. Simulation of fatigue crack growth.

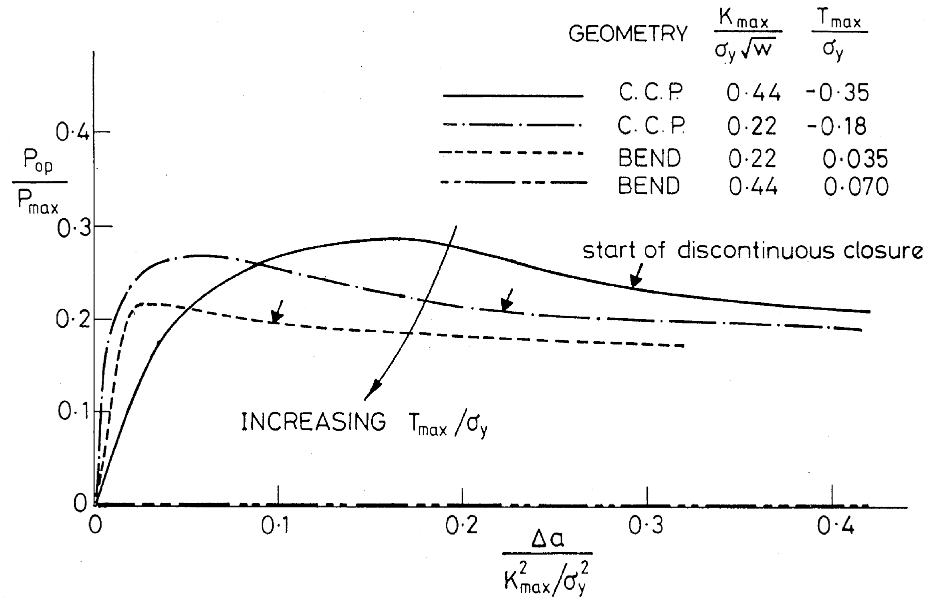

Fig. 3. Effect of load level and specimen geometry on closure response. $R = 0$.

the ASTM limit for small scale yielding. The ratio T_{max}/σ_y for each geometry and load level is included in the figure. The load at which the crack closes, P_c, is not presented since it is highly sensitive to the residual deformation left in the crack tip element, and thereby to the mesh size. It is argued in Fleck and Newman [1] that the mesh is sufficiently fine for the analysis to give representative values for P_{op}/P_{max}.

It appears from Fig. 3 that P_{op}/P_{max} reaches a maximum and then decays in value, with increasing crack advance, $\Delta a/(K^2_{max}/\sigma^2_y)$. With the exception of the early period of growth in the CCP geometry at $K_{max} = 0.44\ \sigma_y\sqrt{w}$, the amount of closure decreases with increasing T_{max}/σ_y. (The anomolously small initial amount of closure in the CCP geometry at $K_{max} = 0.44\ \sigma_y\sqrt{w}$, is associated with crack advance through a coarse part of the mesh, before the crack tip has reached the fine part of the mesh [1].)

For the bend specimen at $K_{max} = 0.44\ \sigma_y\sqrt{w}$, the crack closes over only the first element behind the crack tip. It is argued that closure over this single element dimension is an artefact of the discretisation process, and so no closure is assumed. This assumption was validated as follows. A crack in the bend specimen was grown over 20 small element sizes, with $K_{max} = 0.44\ \sigma_y\sqrt{w}$ and $K_{min} = 0$. The crack was then cycled 10 times with the same K_{max} and K_{min}, while the crack

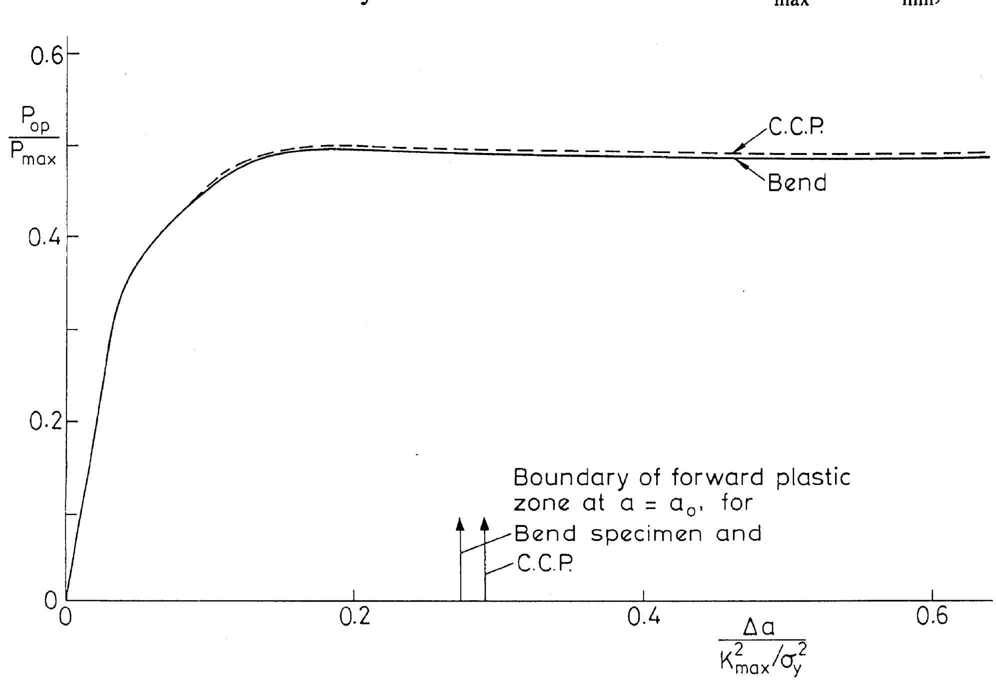

Fig. 4. Crack opening response under plane stress conditions. $K_{max} = 0.157\ \sigma_y\sqrt{w}$, $R = 0$. Crack shuts continuously from tip to near the initial location, a_0, of the crack tip.

length was held fixed. The crack opening load for the element behind the crack tip decreased from 0.19 P_{max} to zero during these 10 cycles. This suggests that closure over a single element dimension is an artefact of the finite element discretisation and should be ignored.

Plane stress results for $R = 0$ are presented in Fig. 4 for comparison with the plane strain results (Fig. 3). Under plane stress conditions, P_{op}/P_{max} increases to a steady state value of 0.50, independent of specimen geometry, that is T_{max}/σ_y. Similar results have been reported by Newman [5].

An examination of the crack opening response showed that, under plane stress conditions, the crack closes *continuously* from its tip to near the initial crack length, a_0. For the case of plane deformations, the nature of the closure process changes from continuous to discontinuous after a sufficient increment of crack growth (Fig. 3). Discontinuous closure is the phenomenon whereby the crack first shuts at a location far from the crack tip. This is shown in Fig. 5a, b for the bend and CCP geometries, where $K_{max} = 0.22\,\sigma_y\,\sqrt{w}$ and $\Delta a = 0.32\,K^2_{max}/\sigma^2_y$. Excluding the node immediately behind the crack tip, the first node to close and the last to open lie close to a_0. Similar behaviour was also observed for the CCP geometry, for $K_{max} = 0.44\,\sigma_y\,\sqrt{w}$.

The source of the discontinuous closure in Fig. 5b appears to be a residual wedge of material on the crack flanks, located just ahead of the initial position of the crack tip. This wedge is formed as the crack evolves from the state of a stationary crack to the steady state of a growing fatigue crack, as depicted in Fig. 6. In order to determine whether P_{op}/P_{max} decays to zero as the crack grows away from this residual wedge of material, a crack was grown in the CCP specimen at $K_{max} = 0.22\,\sigma_y\,\sqrt{w}$, using the following boundary conditions along the crack. The crack flanks were allowed to overlap and not induce closure stresses at any location more than 5 element sizes

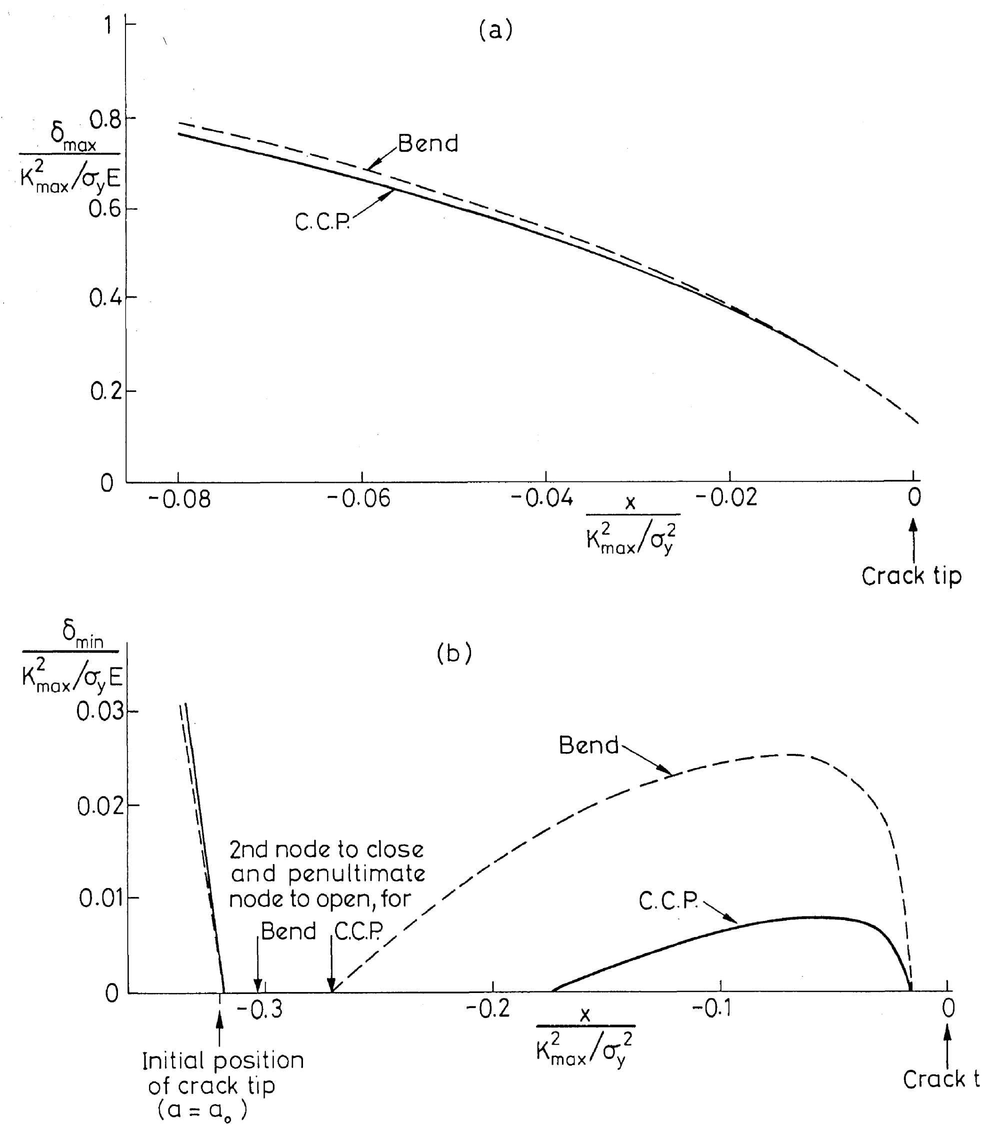

Fig. 5. Crack opening at (a) K_{max} and at (b) K_{min} of a growing fatigue crack. $K_{max} = 0.22\,\sigma_y\,\sqrt{w}$, $K_{min} = 0$, 20 growth steps ($\Delta a = 0.32\,K^2_{max}/\sigma^2_y$).

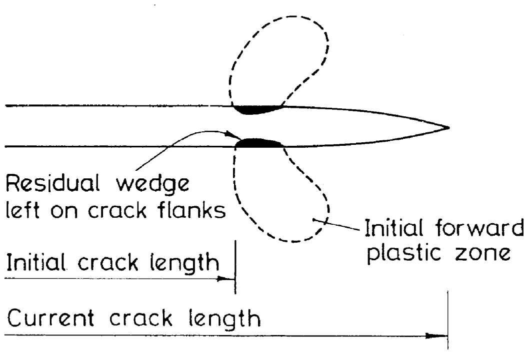

Fig. 6. Schematic showing the nature of the closure phenomenon under plane strain conditions. A residual wedge is left on the crack flanks, as the crack grows through the initial forward plastic zone associated with the initial crack length.

from the crack tip. Crack closure stresses were allowed to develop over the 5 element sizes immediately behind the crack tip. Results are given in Fig. 7, for a crack advance, Δa, of 0.480 K^2_{max}/σ^2_y (30 small element sizes). For this and greater values of Δa the crack opened and closed at zero load to within the accuracy of the analysis. It is deduced from this result that P_{op}/P_{max} asymptotes to zero as Δa tends to infinity, when closure stresses are allowed to develop everywhere along the crack flanks. Fleck and Newman [1] have used a similar procedure to show that P_{op}/P_{max} tends to zero with crack advance, for the case of the CCP geometry when $K_{max}=0.44\ \sigma_y\ \sqrt{w}$. It is thought that this is a general result for the CCP and bend geometries, loaded at $R = 0$ under conditions of small scale yieldings.

The maximum size of the residual wedge of material displayed in Fig. 7 is 0.12 $K^2_{max}/\sigma_y E$, which is comparable with the crack tip opening displacement at K_{max} (Fig. 5a). The size of this residual wedge of material is a lower bound estimate for the following reason. When closure is allowed to occur over a limited number of elements behind the crack tip, extra crushing is experienced by these elements during unloading of the fatigue crack.

3.1. *Plastic zone distribution for a growing fatigue crack*

The plastic zone distribution at $K_{max} = 0.22\ \sigma_y\ \sqrt{w}$ and at $K_{min} = 0$ is given in Fig. 8a and b, for the bend and CCP specimens after a crack advance, Δa, of 0.32 K^2_{max}/σ^2_y. These solutions are qualitatively the same as those discussed in Fleck and Newman [1], where $K_{max} = 0.44\ \sigma_y\ \sqrt{w}$, $K_{min} = 0$.

The nature of the plastic zone distributions may be understood by recalling that a secondary, active yield zone exists along the flanks of a tearing crack, with crack advance occuring at constant $K = K_{max}$. For both the tearing crack and the growing fatigue crack, the stress state in this secondary yield zone is predominantly $\sigma_{xx} \approx \sigma_y$, $\sigma_{yy} \approx 0 \approx \tau_{xy}$. This yield zone exists in order

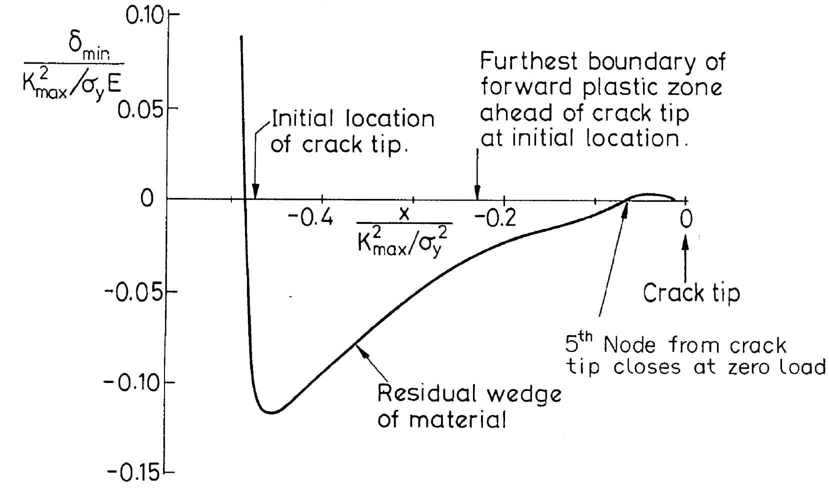

Fig. 7. Crack profile for growing crack at K_{min}. Crack has advanced by $\Delta a = 0.480\ K^2_{max}/\sigma^2_y$, under $K_{max} = 0.22\ \sigma_y\ \sqrt{w}$, $K_{min} = 0$. Crack is allowed to close over only first five elements behind crack tip.

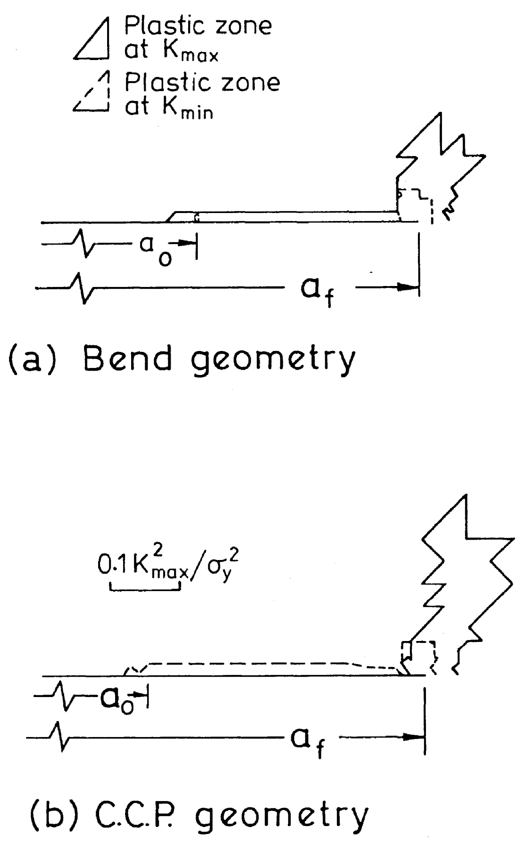

Fig. 8. Active plastic zones at K_{max} and at K_{min} for growing fatigue crack in (a) Bend specimen and (b) CCP geometry. Crack has grown by $\Delta a = 0.32\ K_{max}^2/\sigma_y^2$ under $K_{max} = 0.22\ \sigma_y\sqrt{w}$, $K_{min} = 0$.

to reduce the strains to an elastic order of magnitude in the wake of the advancing crack tip plastic zone [1].

Consider the plastic zone associated with a growing fatigue crack in the bend specimen (Fig. 8a). At K_{max}, the plastic zone distribution is similar to that of a tearing crack [1]. The T-stress is positive and reinforces the active plastic zone alongside the crack flanks. Upon unloading to K_{min}, the change in T-stress is negative and material beside the crack flanks unloads elastically (Fig. 8a). A reversed plastic zone exists at the crack tip and a single element has yielded at a_0 due to indentation of one crack flank into the other.

Now consider the CCP geometry. At $K = K_{max}$ (Fig. 8b), the T-stress is negative and no secondary plastic zone exists behind the crack tip. Upon unloading to $K_{min} = 0$, the change in T-stress is positive and the secondary plastic zone along the crack flanks reappears (Fig. 8b). The stress state in this secondary zone is again $\sigma_{xx} \approx \sigma_y$, $\sigma_{yy} \approx 0 \approx \tau_{xy}$.

For both geometries, the plastic zone ahead of the fatigue crack tip at K_{max} is similar to that of a tearing crack under constant $K = K_{max}$ [1]. At K_{min}, a reversed plastic zone exists at the crack tip (Fig. 8a and b); this is similar in size to the reversed plastic zone at the tip of a stationary crack [1].

3.2. *Influence of material properties and loading history on closure behaviour*

Results have already been given for $\sigma_y/E = 1/200$, $\nu = 0.3$, which are typical properties of a high strength aluminium alloy. The closure response for $\sigma_y/E = 1/600$ and $\nu = 0.3$, properties typical of a medium strength steel, are compared with these previous results in Fig. 9. The effect of elastic compressibility is also examined, by determining the crack closure behaviour for an almost incompressible material with $\nu = 0.49$, and $\sigma_y/E = 1/200$. In each case, the CCP geometry is considered, with $K_{max} = 0.22\ \sigma_y\sqrt{w}$ and $K_{min} = 0$.

It appears from Fig. 9 that the crack closure response is insensitive to a change of σ_y/E or ν. In all cases, P_{op}/P_{max} attains a maximum of approximately 0.25 after a crack growth increment, $\Delta a/(K_{max}^2/\sigma_y^2)$, of about 0.05. Thereafter, P_{op}/P_{max} decays in magnitude with further crack advance. Since plasticity-induced crack closure occur even for $\nu = 0.49$, it is deduced that the residual

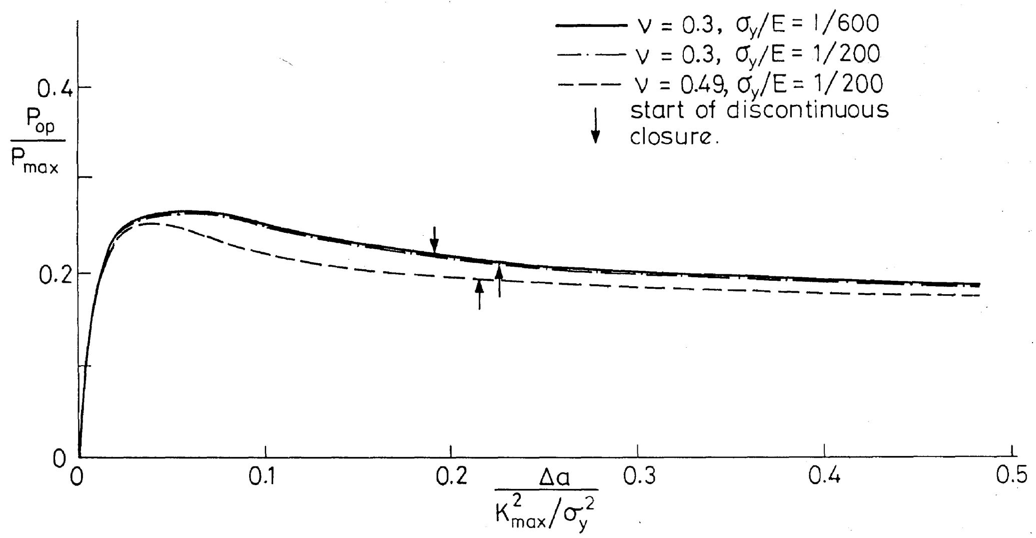

Fig. 9. Effect of material properties on the closure transient, for CCP geometry. $K_{max} = 0.22\ \sigma_y\sqrt{w}$, $K_{min} = 0$.

wedge of material on the crack flanks does not originate from a dilatation of the remote elastic field.

Experimental studies of plasticity-induced crack closure under plane strain conditions suggest that at $R = 0$ the ratio P_{op}/P_{max} decreases from about 0.2 to zero with increasing yield stress, for steels and aluminium alloys [7]. The results presented in Fig. 9 suggest that this is not due to an increase in σ_y/E with increasing σ_y. It is thought that the observed decrease in closure with increasing yield stress is due to a decrease in strain hardening capacity and to an increase in the degree of cyclic softening with increasing yield stress. Further work is required to resolve this issue.

The influence upon P_{op}/P_{max} of a gradual increase in K_{max} with crack extension is shown in Fig. 10. The crack opening load builds-up more slowly with crack extension, for the case where K_{max} is increased gradually to $0.22\ \sigma_y\sqrt{w}$ than for the case where K_{max} is held constant at $0.22\ \sigma_y\sqrt{w}$. In qualitative terms the transients are similar: P_{op}/P_{max} increases to a maximum of approximately 0.25 and then decreases with increasing crack advance. It is concluded that it is very difficult to eliminate the closure transient associated with the evolution of a crack from the state of a stationary crack to the state of a growing fatigue crack.

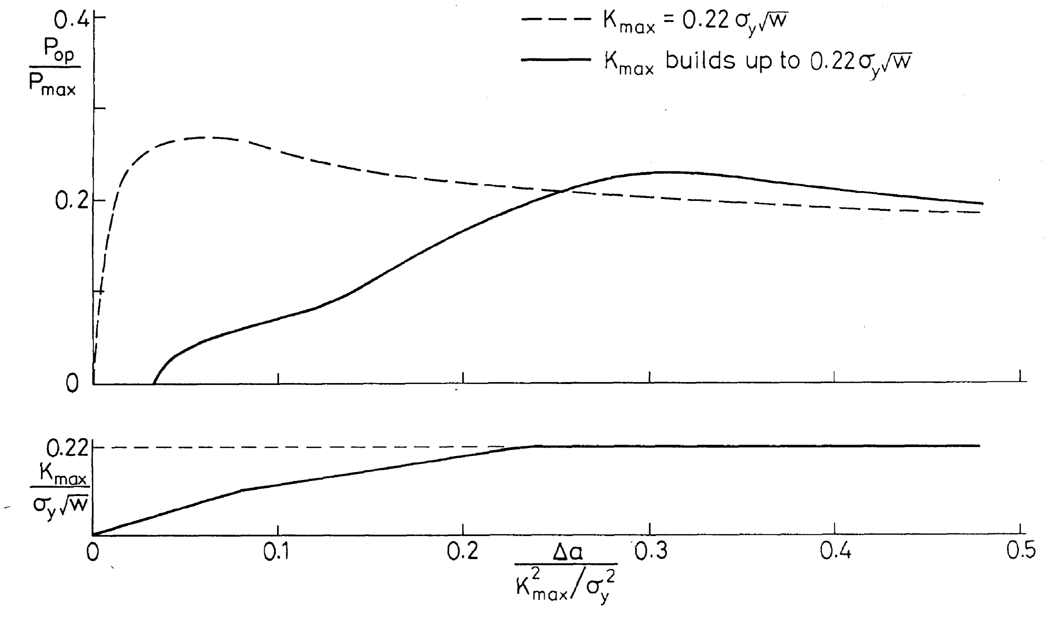

Fig. 10. Influence of K-gradient on the closure response, for CCP geometry. $K_{min} = 0$, final $K_{max} = 0.22\ \sigma_y\sqrt{w}$.

4. CONCLUSIONS

Fatigue crack growth under plane strain conditions has been idealised by a finite element analysis of an elastic–perfectly plastic solid. A load ratio, R, of zero was considered, and the following conclusions have been drawn.

Plasticity-induced crack closure occurs over a transient increment of crack growth, provided the ratio T_{max}/σ_y is less than a critical value in the range 0.035–0.070. The T-stress is the non-singular contribution to the normal stress acting parallel to the crack flanks near the crack tip. It is used successfully in the present study to account for the influence of specimen geometry and load level upon crack closure response.

The analysis suggests that crack closure occurs as a crack evolves from the state of stationary crack to the steady state of a growing fatigue crack. The ratio of crack opening load to maximum load of the fatigue cycle, P_{op}/P_{max}, rises to a maximum of approximately 0.25 after the crack has grown a distance $\Delta a/(K_{max}^2/\sigma_y^2)$ of 0.05–0.15. Thereafter, P_{op}/P_{max} decays to zero. A residual wedge of material is left on the crack flanks at a location immediately ahead of the initial location of the crack tip. This wedge leads to discontinuous closure of the crack.

The plastic zone distribution ahead of a growing fatigue crack bears a similarity to both a stationary crack and a tearing crack. An active plastic zone exists along the crack flanks at K_{max} if $T\sqrt{w}/K$ is positive, and at K_{min} if $T\sqrt{w}/K$ is negative. The reversed plastic zone ahead of the crack tip is similar to that of a stationary crack.

The crack closure response, P_{op}/P_{max}, is sensitive to the amount of crack advance, $\Delta a/(K_{max}^2/\sigma_y^2)$, and to the ratio T_{max}/σ_y, but is insensitive to the material properties σ_y/E and ν.

Plasticity–induced closure under plane stress conditions is quite different from the behaviour described above for plane deformations. When plane stress conditions prevail, P_{op}/P_{max} increases to a stabilised value with crack advance; this steady-state value is much higher than the peak of the P_{op}/P_{max} versus $\Delta a/(K_{max}^2/\sigma_y^2)$ curve for plane strain conditions. Under plane stress conditions the crack always opens and closes in a continuous manner, whereas under plane strain conditions the crack exhibits discontinuous closure after some crack growth.

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