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Adiabatic Shear Instability: Theory

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Introduction

At sufficiently high strain rates, many polymers undergo localised shear deformation. This material instability is due to *thermal softening* dominating *strain hardening*. Data are sparse in comparison with shear localisation in metals, notably titanium alloys and high strength steels. A summary of theoretical understanding of the subject is now given, with references to experimental data as appropriate.

Isothermal versus adiabatic tests

The thermal diffusivity of polymers is low ($\kappa \approx 10^{-7} \text{ m}^2\text{s}^{-1}$) when compared with that for metals ($\kappa \approx 10^{-4} \text{ m}^2\text{s}^{-1}$). Consequently, material tests switch from isothermal to adiabatic at much lower strain rates for polymers than for metals. Consider a test done in simple shear on a polymer specimen of length scale L . Plastic deformation of the specimen results in a temperature rise, and the time t required for heat to diffuse from the centre of the specimen to the environment is given approximately by the random walk equation,

$$L \approx \sqrt{\kappa t} \quad (1)$$

The time of the test can be taken to be $t \approx 1/\dot{\gamma}$, for a test at constant shear strain rate $\dot{\gamma}$. Thus, the heat generated within the sample is dissipated adequately (the test is *isothermal*) provided the strain rate $\dot{\gamma}$ satisfies

$$\dot{\gamma} < \frac{\kappa}{L^2} \quad (2)$$

On assuming representative values, $L = 10 \text{ mm}$ and $\kappa \approx 10^{-7} \text{ m}^2\text{s}^{-1}$ for a polymer, we find that the test is isothermal provided $\dot{\gamma} < 10^{-3} \text{ s}^{-1}$. At higher strain rates the test is adiabatic. Typical quasi-static engineering tests on polymers are conducted at strain rates in the range $10^{-5} - 10^{-3} \text{ s}^{-1}$, and so can be regarded as isothermal. *Adiabatic* tests are performed at higher strain rates (up to 10 s^{-1} in a servo-hydraulic test machine; about 10^2 s^{-1} in an instrumented drop weight machine, about 10^3 s^{-1} in a split-Hopkinson bar and at about 10^4 s^{-1} in a plate impact test).

In these adiabatic tests the specimen heats up significantly during the test due to internal plastic dissipation. Since the flow strength τ of polymers decreases with increasing temperature, the adiabatic temperature increase leads to *thermal softening* and to the possibility of the unstable growth of a localised shear band.

The temperature increase ΔT due to plastic dissipation can be estimated in a straightforward manner by equating the plastic work per unit volume $\int \tau d\gamma$ with the increase in internal energy $\rho c \Delta T$. For example, a polymer such as PMMA or PC has a shear strength τ of approximately 50 MPa, a density of $\rho = 1000 \text{ kgm}^{-3}$, and a specific heat capacity $c = 1300 \text{ J kg}^{-1} \text{ K}^{-1}$. Then, on imposing a shear strain of $\gamma=1$, the computed temperature rise ΔT is about 38 K. This result has two implications:

- (i) the temperature in an adiabatic test changes significantly over the test and the measured stress-strain response is softer than that in an isothermal test at the same strain rate;
- (ii) the progressive thermal softening in an adiabatic test can lead to shear localisation, whereas in the equivalent isothermal test the response is stable. We demonstrate below that instability initiates at the point of maximum shear stress in a shear test.

A comparison of the isothermal and adiabatic responses is sketched in Fig. 1 to demonstrate the thermal softening associated with an adiabatic test. When the degree of thermal softening is sufficient the adiabatic test shows a maximum in shear stress and an instability, whereas the isothermal test shows continued strain hardening and a stable response.

Stability analysis: the onset of localisation

The onset of shear localisation can be predicted by imposing a perturbation about the current state in a stress-strain test, and determining whether the perturbation grows unstably^{1,2}. Consider a shear test with the shear stress related to the shear strain γ , and to the current temperature T in the material by

$$\tau = f(\gamma, T) \tag{3}$$

Then, a perturbation in (γ, T) results in a shear stress perturbation $\delta\tau$ of

$$\delta\tau = \frac{\partial f}{\partial \gamma} \delta\gamma + \frac{\partial f}{\partial T} \delta T \tag{4}$$

But, in an adiabatic test, an increment in shear strain $\delta\gamma$ results in a temperature increase of

$$\delta T = \frac{\tau}{\rho c} \delta\gamma \quad (5)$$

and so relation (4) can be simplified to

$$\delta\tau = \left(\frac{\partial f}{\partial \gamma} + \frac{\partial f}{\partial T} \frac{\tau}{\rho c} \right) \delta\gamma \quad (6)$$

The instability grows when the perturbation $\delta\gamma$ results in a negative value of $\delta\tau$, and so the onset of instability in an adiabatic test occurs according to the criterion:

$$\frac{\partial f}{\partial \gamma} + \frac{\partial f}{\partial T} \frac{\tau}{\rho c} = 0 \quad (7)$$

This corresponds to the peak value of shear stress in an adiabatic test. The criterion (7) remains valid in the presence of strain rate hardening, and when inertial and thermal conductivity are included in the analysis (2). It simply states that **adiabatic shear localisation occurs when the rate of thermal softening outweighs the strain hardening rate**. Rapid thermal softening occurs in the vicinity of the glass transition temperature T_g for amorphous polymers (and in the vicinity of the softening temperature for semi-crystalline polymers). Thus shear localisation occurs when the initial temperature in an adiabatic test is in the vicinity of T_g . But shear localisation can also occur at lower temperatures, brought about by the negative strain hardening rate induced by crazing and microcrack formation. This mechanism has been identified by Fleck, Stronge and Liu (3), but does not appear to be appreciated within the general literature.

The width of a shear band

The prediction of the width w of a shear band is not fully resolved. There are at least two viewpoints on the main controlling factors for the shear band width:

(i) the width w is set by the distance over which heat can diffuse during the formation of a shear band (4). Here, it is assumed that the shear band width is fixed by the transient period of shear band nucleation and growth. On taking the formation time t to be approximately, $t \approx 1/\dot{\gamma}$, heat can diffuse over a distance $w \approx \sqrt{\kappa t}$ on making use of relation (1), and so w is given by

$$w \approx \sqrt{\kappa / \dot{\gamma}} \quad (8)$$

Substitution of typical values for $\dot{\gamma} = 10^3 \text{ s}^{-1}$ in a high strain rate test, and $\kappa \approx 10^{-7} \text{ m}^2 \text{ s}^{-1}$ for a polymer, gives $w \approx 10 \text{ }\mu\text{m}$. This value is of the same order of magnitude as the measured value.

(ii) the width w is set by the distance over which heat is conducted in steady state, after all transients have finishes (5). Consider a simple heat flow balance for a band of material of width w wherein plastic dissipation occurs at a constant rate $\tau\dot{\gamma}$. The temperature at the centre of the band is taken to be elevated by ΔT above that of the material outside the band. On assuming steady state conditions, this power is dissipated by thermal conduction across the band boundary, giving

$$w\tau\dot{\gamma} \approx \lambda \frac{\Delta T}{w} \quad (9)$$

where λ is the thermal conductivity of the polymer. Thus, the band width w is given by $w \approx \sqrt{(\lambda\Delta T / \tau\dot{\gamma})}$. Note that for steady state conditions to be attained, this band width should be stable with respect to time, and neither increase or diminish in an unstable manner: an additional stability statement is required in addition to the satisfaction of (9). Details are omitted here, but the additional statement reads

$$\left(\frac{w\tau\dot{\gamma}}{\lambda}\right)^2 \frac{\partial^2 f}{\partial T^2} + \frac{2\dot{\gamma}}{w^2} \frac{\partial f}{\partial \dot{\gamma}} + \left(\frac{\dot{\gamma}}{w}\right)^2 \frac{\partial^2 f}{\partial \dot{\gamma}^2} > 0 \quad (10)$$

where the constitutive law for the solid is written in the form $\tau = f(\dot{\gamma}, T)$. The practical relevance of relation (9) is questioned: shear band formation is a highly transient phenomenon, and steady state conditions are rarely achieved in practice.

References

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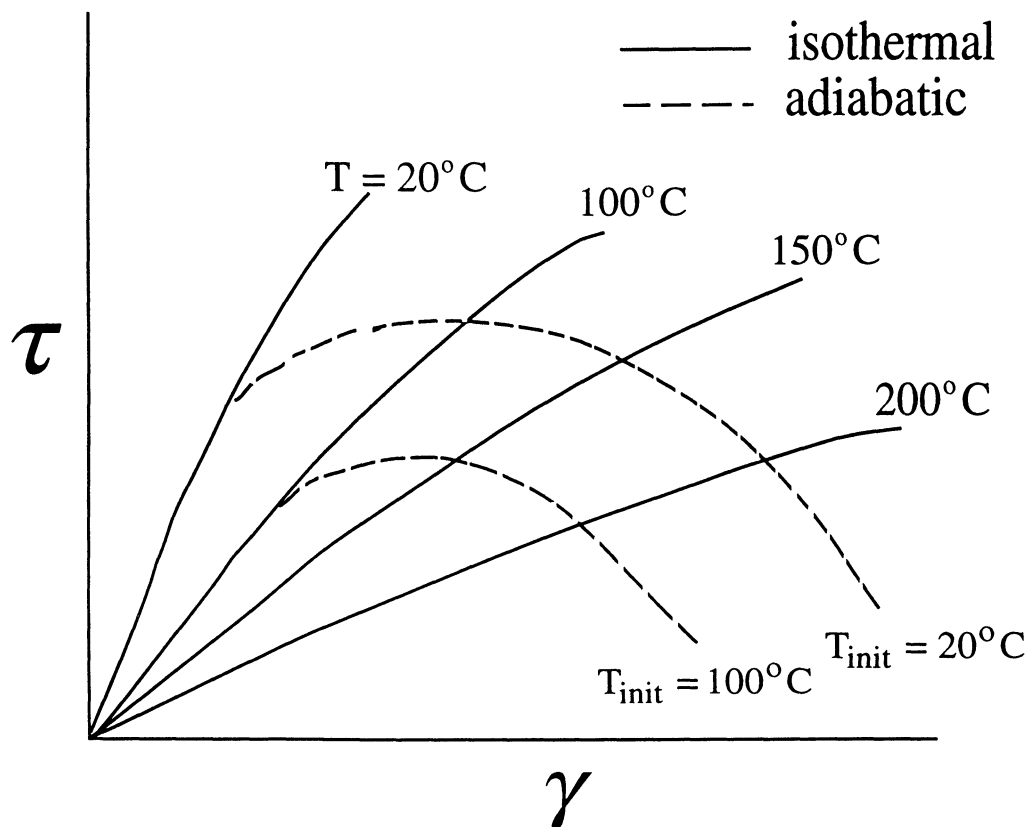


Fig. 1. Comparison of isothermal and adiabatic shear responses at fixed strain rate. Thermal softening in the adiabatic test can lead to a maximum load and to shear localisation.