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The out-of-plane compressive behaviour of woven-core sandwich plates

M. Zupan, V.S. Deshpande, N.A. Fleck

Department of Engineering, University of Cambridge, Trumpington St., Cambridge, CB2 1PZ, United Kingdom

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Abstract

Stainless steel woven-core sandwich plates of length L, height H and width W have been manufactured by brazing together commercially available wire meshes of cell size l. The out-of-plane compressive response of the sandwich plate is measured for three relative densities of the woven core, and the effects of the plate aspect ratios L/H, width to height ratio W/H and height to cell diagonal ratio H/d are investigated. When the plate is thin such that L/H exceeds about 10, the compressive strength is insensitive to the value of L/H. Conversely, for $L/H \leqslant 10$, the compressive strength drops sharply with decreasing value of L/H. Analytical models are presented for the dependence of the compressive stiffness and strength of the woven cores upon the core relative density and plate aspect ratio L/H. The models predict that the stiffness and strength are dictated by axial stretch of the individual struts, and scale linearly with the relative density. Good agreement is observed between the measured and predicted stiffness and peak strengths.

Keywords: Textile materials; Cellular solids; Plastic buckling; Sandwich plate

1. Introduction

Sandwich panels comprising stiff and strong faces and a low density core are often used in weight-efficient structures. In the past few years a number of sandwich core topologies have been proposed, see for example the reviews of Evans et al. (2001) and Wadley et al. (2003). In broad terms these cores can be sub-divided into *stochastic cellular solids*, such as metallic foams, and *periodic cellular solids* including the hexagonal honeycomb and prismatic diamond. The effective mechanical properties of both periodic and stochastic open-cell microstructures are dictated largely by the nodal connectivity at the junctions between struts. For example, the octet truss has a sufficiently large nodal connectivity of 12 that individual struts deform by stretching rather than by local bending, Deshpande et al. (2001). Consequently, the stiffness and strength of lattice materials scale linearly with their relative density $\bar{\rho}$. A two dimensional variant of lattice material is the *woven textile*, manufactured by brazing together commercially available wire meshes to produce textile core, as introduced by Sypeck and Wadley (2001). A photograph of grade 316 stainless steel wire mesh, as used in the present investigation, is shown in Fig. 1(a). The wire mesh is characterized by a wire radius a, cell size ℓ, weave angle ω, and cell diagonal length $d = 2l\sin\omega$ as sketched in Fig. 1(b). (Only orthogonal weaves with $\omega = 45^\circ$ are used in this study.) Identical wire meshes are stacked layer by layer and bonded, with the cells aligned to give prismatic diamond-like cells along the stacking direction, see Fig. 2. Stainless steel face sheets are then bonded to the sides of the stack in order to produce a sandwich plate of length L, height H and width W. The relative density $\bar{\rho}$ of the textile core is defined as the ratio of the core density to that of the solid. Upon neglecting the added mass due to the braze material, straightforward geometrical considerations give

$$\bar{\rho} = \frac{\pi}{2\sin 2\omega}\left(\frac{a}{\ell}\right). \tag{1.1}$$

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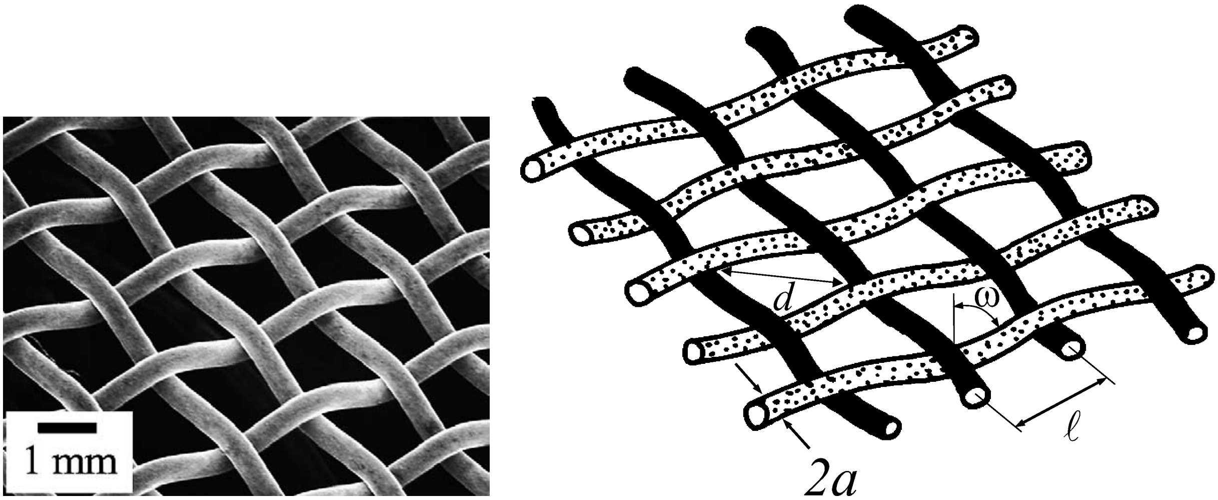

(a) (b)

Fig. 1. (a) Photograph of 316 stainless steel wire mesh used to manufacture the woven cores; (b) Sketch of the wire mesh of wire radius a, cell diagonal length d, cell size ℓ and weave angle ω.

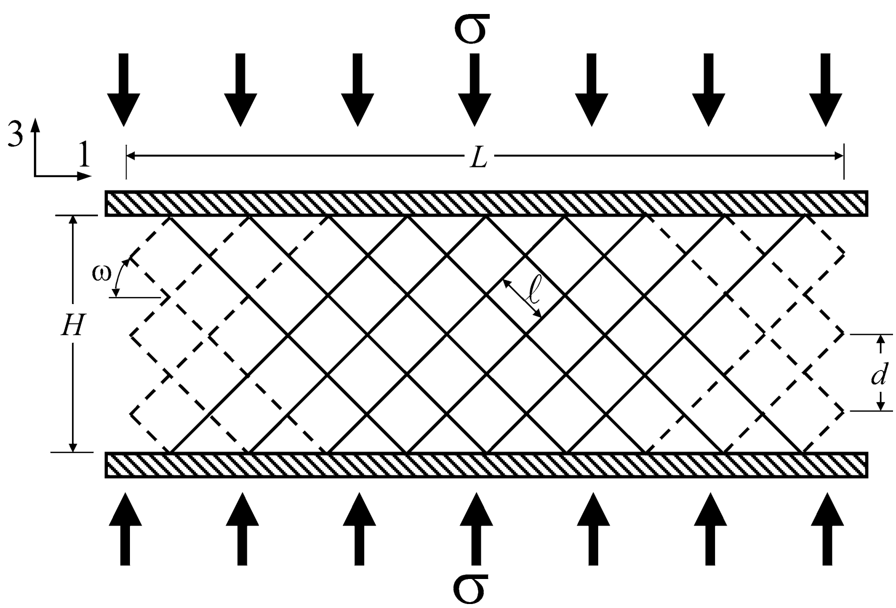

Fig. 2. Sketch of a woven core sandwich plate under out-of-plane compression. The solid lines are mesh wires that are bonded to both face sheets and carry load while the dashed lines are mesh wires that are bonded to one face sheet only and are assumed to carry no load in the lower bound analysis.

The aim of the current study is to investigate experimentally and analytically the out-of-plane compressive response of a sandwich panel with a woven textile core. First, analytical models are reviewed for the out-of-plane stiffness, yield strength and peak strength of a sandwich plate with a woven core. Second, the manufacturing method is described for a stainless steel sandwich plate with a woven core. Third, the compressive response is measured for three relative densities of the woven core, and finally the measured stiffness and peak strengths are compared with analytical predictions.

2. Review of the out-of-plane effective properties of the woven-core sandwich plate

Approximate expressions are reviewed for the out-of-plane modulus E and the compressive yield and peak strengths σ_Y and σ_P, respectively, of the woven core by neglecting the effects of the waviness of the mesh wires. The full derivation of the formulae are presented in Cote et al. (2004a, 2004b); they also show that the knock-down in strength due to wire waviness is small and can be neglected for most practical purposes.

2.1. Elastic properties

Consider a sandwich plate of length L with rigid face-sheets and woven core of thickness H as sketched in Fig. 2. The woven core comprises mesh wires that are attached to both face sheets (shown by solid lines in Fig. 2) and wires that are only attached to one face sheet (dashed lines). Upon employing a lower bound approach, it is assumed that wires attached to only one face sheet carry no load while all other wires are subject to an axial stress σ_f, in equilibrium with a macroscopic applied stress σ. The following energy calculation is used to deduce the effective elastic modulus of the core in the out-of-plane direction. The elastic strain energy in N load-carrying wires of the core of width $4a$ (which is the width of one layer of the weave) is given by

$$\frac{1}{2}\frac{\sigma^2}{E}4aLH = \frac{1}{2}\frac{\sigma_f^2}{E_s}N\pi a^2\frac{H}{\sin\omega}, \tag{2.1}$$

where E and E_s are the macroscopic modulus of the woven core and the Young's modulus of the weave material, respectively. The number of load carrying wires N is given by

$$N = \frac{(L - H/\tan\omega)}{\ell\cos\omega}. \tag{2.2}$$

Equilibrium in the out-of-plane direction (the 3-direction of Fig. 2) gives the relation between the macroscopic stress σ and the wire stress σ_f as

$$\sigma = N\frac{\sigma_f\pi a^2\sin\omega}{4aL}. \tag{2.3}$$

Upon substituting for N and σ from (2.2) and (2.3), respectively, into (2.1) we obtain

$$\frac{E}{E_S} = \frac{\pi}{4}\left(\frac{a}{\ell}\right)\left(1 - \frac{1}{A\tan\omega}\right)\frac{\sin^3\omega}{\cos\omega} \equiv \bar{\rho}\left(1 - \frac{1}{A\tan\omega}\right)\sin^4\omega, \tag{2.4}$$

where the aspect ratio of the plate A is defined by $A = L/H$.

2.2. Yield strength

The out-of-plane compressive yield strength of the woven core can be determined directly from equilibrium. The woven wires are taken as elastic-ideally plastic, with a yield strength Y, and the relation between the macroscopic stress σ and the fibre stress σ_f is again given by (2.3). Upon equating the axial stress within the wires σ_f to the yield strength Y of the parent material, the macroscopic yield strength of the weave σ_Y is

$$\frac{\sigma_Y}{Y} = \frac{\pi}{4}\left(\frac{a}{\ell}\right)\left(1 - \frac{1}{A\tan\omega}\right)\tan\omega \equiv \bar{\rho}\sin^2\omega\left(1 - \frac{1}{A\tan\omega}\right). \tag{2.5}$$

2.3. Plastic buckling

For woven materials made from a strain hardening solid, the woven core continues to carry load beyond first yield as given by (2.5). The peak compressive strength is set by plastic buckling of the individual cell members of the woven core, with the peak macroscopic stress σ_P evaluated by replacing Y in Eq. (2.5) with the plastic bifurcation stress σ_c for a strut of length ℓ. The stress σ_c is given by the Shanley tangent-modulus buckling formula

$$\sigma_c = \frac{k^2\pi^2E_ta^2}{4\ell^2}, \tag{2.6}$$

where E_t is the tangent-modulus, defined by the slope $d\sigma/d\varepsilon$ of the uniaxial true stress versus logarithmic strain curve of the solid material at the stress level σ_c. The factor k in Eq. (2.6) depends upon the rotational stiffness of the end nodes of the strut. The lowest buckling mode of the woven sandwich core under uniaxial compression in the 3-direction is sketched in Fig. 3; this mode corresponds to struts of length ℓ buckling as pin-ended struts and thus $k = 1$ in (2.6).

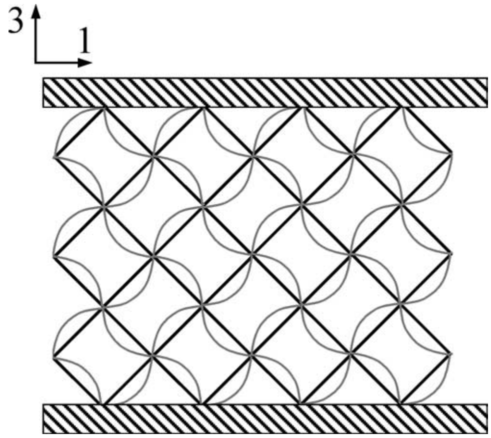

Fig. 3. Sketch of the assumed plastic buckling mode (gray lines) of the wire mesh.

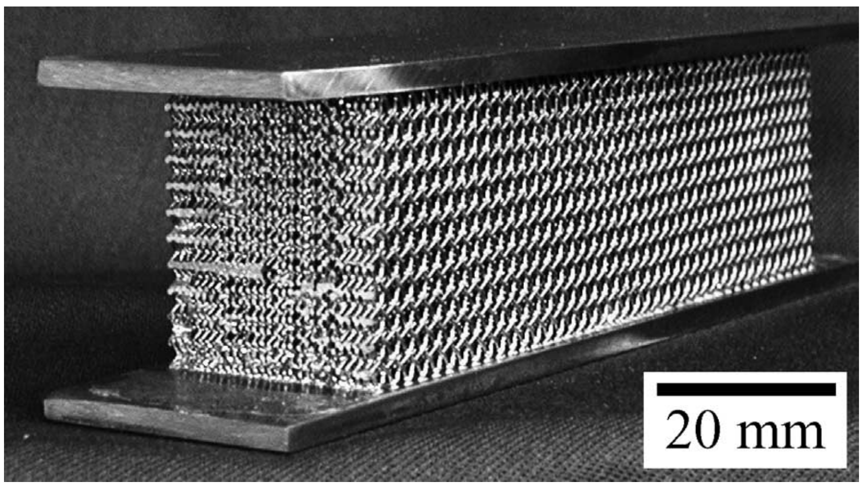

Fig. 4. Photograph of a sandwich plate with a woven core of relative density $\bar{\rho} = 0.23$ manufactured from 316 stainless steel wire mesh.

3. Manufacture of textile cores

Textile-core sandwich plates were manufactured by brazing together a stack of commercially available 316 stainless steel wire meshes, as suggested by Sypeck and Wadley (2001). We briefly describe the manufacturing route here: additional details are contained in Sypeck and Wadley (2001).

A stack of wire mesh was adhered together with polymer cement and braze filler[1] of nominal composition Ni–Cr 25–P10 (wt.%). Brazing was conducted at 1120 °C in a dry inert environment of argon at 2–8 × 10^{-2} Torr. Specimens of length L and height H were then spark machined from the brazed woven plates of width W such that the mesh wires made an angle of $\pm 45^\circ$ with respect to the specimen edges, as shown in Fig. 2. Finally, grade 316 stainless steel face-sheets of thickness 2.5 mm were bonded to the top and bottom of the edges using a second brazing cycle. The second braze cycle repeats the first with the exception that the brazing was conducted at 1030 °C: the diffusion of phosphorous out of the braze filler into the surrounding stainless steel in the first brazing cycle increases the braze melting temperature to above 1030 °C. Consequently, in the second braze cycle, the face sheets were brazed to the core without affecting the already bonded core. A photograph of a woven core specimen with relative density $\bar{\rho} = 0.23$ and aspect ratios $L/H = 4$, $H/d = 10$ and $W/H = 1$ is presented in Fig. 4 for illustrative purposes. The geometry, and measured and predicted values of relative density $\bar{\rho}$ are given in Table 1 for the woven core specimens of weave angle $\omega = 45^\circ$. Good agreement is noted between the predicted and measured values of $\bar{\rho}$.

[1] Wall Colmonoy, Madison Heights, Michigan 48071, USA.

Table 1
Microstructure of woven cores

a (mm)	l (mm)	ω (deg)	$\bar{\rho}$ (measured)	$\bar{\rho}$ (predicted)
0.28	1.92	45	0.230	0.228
0.42	3.32	45	0.200	0.196
0.63	5.50	45	0.180	0.178

Table 2
Specimen geometries

$\bar{\rho}$	L/H	H/d	W/H
	2	5	1
	4	5	1
	8	5	1
	12	5	1
0.23	4	7	1
	4	10	1
	4	15	1
	4	5	0.5
	4	5	2
	1.5	8	1
0.20	2.5	8	1
	5	8	1
	2	5	1
0.18	4	5	1
	8	5	1

4. Measurement protocol

Compression tests were performed in the out-of-plane, 3-direction, of the woven core sandwich plates, as sketched in Fig. 2. A screw-driven test machine was used to apply an average strain rate in the core of $1 \times 10^{-4}\ \mathrm{s}^{-1}$. The relative displacement of the sandwich plate face sheets was measured on opposite sides of the specimen with a pair laser extensometers, and this relative displacement was used to define the nominal strain ε within the core. The applied load was measured using the load cell of the test machine, and was used to define the nominal stress σ on the specimen. In all tests, an unloading reloading cycle was performed after the specimens had been compressed to $\varepsilon \approx 5\%$ in order to measure the unloading modulus of the woven sandwich core.

The purpose of the tests was to determine the sensitivity of the out-of-plane stiffness and strength to (i) the aspect ratio L/H, (ii) the number of cells over the height of the specimen as parameterized by H/d, and (iii) the width of the beam as parameterized by W/H. A summary of the geometries considered is given in Table 2.

5. Experimental results

5.1. *Effect of the aspect ratio L/H upon compressive strength*

Compressive stress versus strain curves of the woven core specimens with $W/H = 1$ are shown in Figs. 5(a)–(c) for $\bar{\rho} = 0.23$, 0.20 and 0.18, respectively. (While the compressive responses of the $\bar{\rho} = 0.23$ and 0.18 specimens have a height to cell size ratio $H/d = 5$, the compressive response of the $\bar{\rho} = 0.20$ specimens have $H/d = 8$.) In each plot results are given for selected values of L/H in the range 1.5 to 8.

The compressive stress versus strain curves shown in Fig. 5 all have the same form with an initial linear regime, followed by a non-linear hardening response, a peak stress at a uniaxial compressive strain of $\varepsilon \approx 6\%$, and a mildly softening response beyond peak stress. Densification of the woven core is observed at strains exceeding approximately 50%, and is associated with a strongly hardening response. In all cases shown in Fig. 5 the initial stiffness and peak strength increase with increasing aspect ratio L/H, but the shapes of the stress versus strain curves are qualitatively similar.

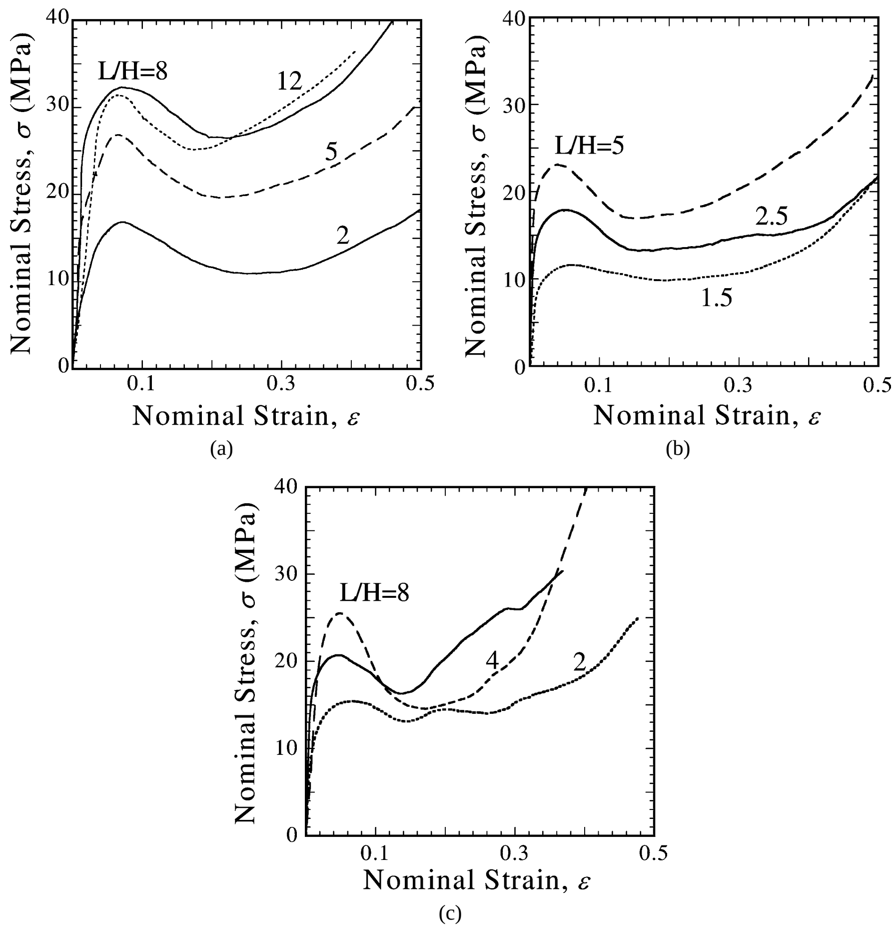

Fig. 5. Nominal compressive stress versus strain response of the woven core sandwich plates (a) $\bar{\rho} = 0.23$ ($H/d = 5$), (b) $\bar{\rho} = 0.20$ ($H/d = 8$) and (c) $\bar{\rho} = 0.18$ ($H/d = 5$). In all cases, the width to height ratio is $W/H = 1$.

5.2. Effect of the ratio of core height to cell size H/d

The dependence of the compressive stress versus strain response upon the ratio of plate height to cell size H/d is shown in Fig. 6 for the specimens of relative density $\bar{\rho} = 0.23$ and aspect ratio $L/H = 4$. The response is relatively insensitive to the height of the core over the range explored, $5 \leqslant H/d \leqslant 16$. Thus, the choices $H/d = 5$ and $H/d = 8$ used above for exploring the effect of aspect ratio L/H upon strength (Fig. 5) are sufficient to ensure minimal boundary layer effects between core and face sheets.

5.3. Effect of the ratio of the core width to height W/H

A side view of a representative specimen ($\bar{\rho} = 0.23$, $L/H = 4$, $H/d = 7$ and $W/H = 1$) compressed to strain $\varepsilon = 40\%$ is shown in Fig. 7. The figure shows that 2–4 weave layers delaminate near the front and rear faces of the woven core sandwich plate. To investigate the significance of this delamination upon the compressive response, we conducted compression tests on woven core plates of core relative density $\bar{\rho} = 0.23$ with different width to height ratios W/H. The results are presented in Fig. 8 for the choices $W/H = 0.5$, 1.0 and 2.0 (with $L/H = 4$ and $H/d = 5$). While the peak compressive strengths of the woven cores with $W/H = 2.0$ and 1.0 are approximately equal, the $W/H = 0.5$ core has a peak strength that is about 30% lower. Thus, the standard choice $W/H = 1$ in this study is sufficient to ensure a negligible effect of delamination of the outer layers upon the compressive strength.

It is noted from Fig. 8 that the specimen of geometry $W/H = 1$ exhibits more pronounced softening beyond the peak stress than the specimens of geometry $W/H = 2.0$ and 0.5. This is most likely related to scatter in the manufacturing technique: the specimens with $W/H = 0.5$ and 2.0 possessed an excess of braze at the nodes, resulting in premature densification.

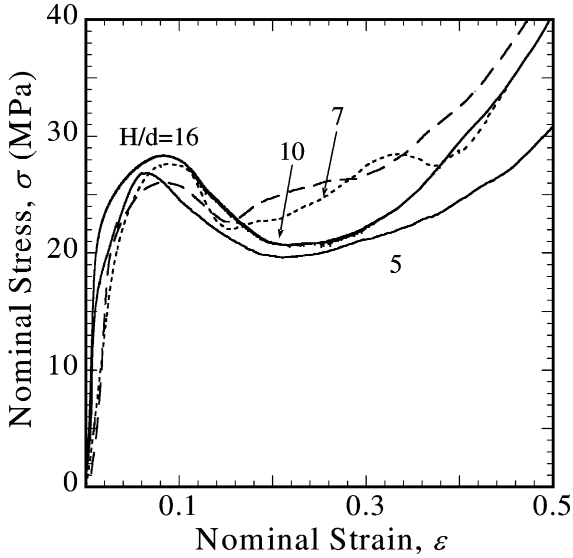

Fig. 6. The effect of specimen height to cell size ratio H/d, on the compressive stress versus strain response. The relative density is $\bar{\rho} = 0.23$, the aspect ratio is $L/H = 4$, and the width to height ratio is $W/H = 1$.

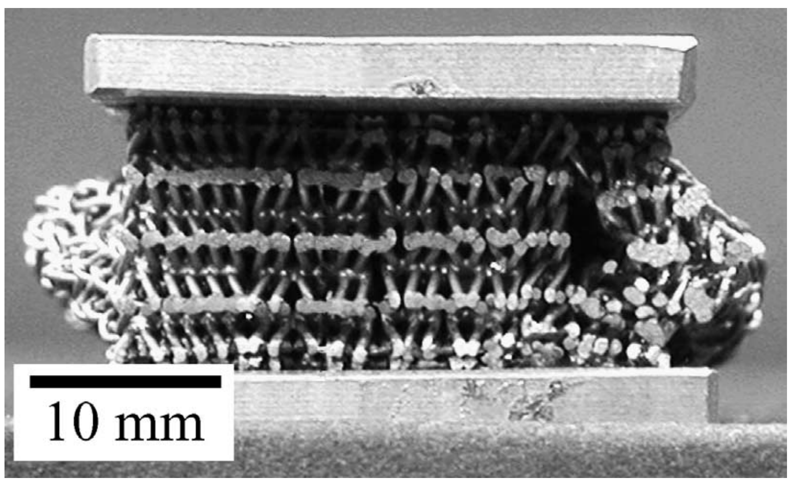

Fig. 7. A side view of a woven core specimen compressed to a strain $\varepsilon \approx 40\%$ ($\bar{\rho} = 0.23$, $L/H = 4$, $H/d = 7$ and $W/H = 1$). The photograph indicates that two to four layers at the front and back faces of the specimen delaminate.

5.4. Material properties of the parent stainless steel

In order to compare the predicted and measured stiffness and strength of the sandwiched woven core, it is first necessary to measure the uniaxial tensile response of the 316 stainless steel in the as-brazed condition. Grade 316 stainless steel sheet in the same metallurgical condition as that of the drawn wire were obtained from the manufacturer of the wire mesh.[2] Tensile specimens of dog-bone geometry and thickness 0.9 mm were cut from the as-received sheet and were subjected to the same brazing cycle as employed in the manufacture of the woven core specimens. The tensile response was measured at a strain rate of $1 \times 10^{-4}\ \mathrm{s}^{-1}$ in accordance with the ASTM standard E8M-96 (1996). The uniaxial tensile true stress versus logarithmic strain curve is shown in Fig. 9, and provides the following material properties: Young's modulus $E_s = 200$ GPa, 0.2% offset yield strength $Y = 200$ MPa and an approximately linear strain hardening response with a linear hardening modulus $E_t = 1.36$ GPa.

[2] Locker Wire Weavers, Warrington Cheshire WA1 2WW, UK.

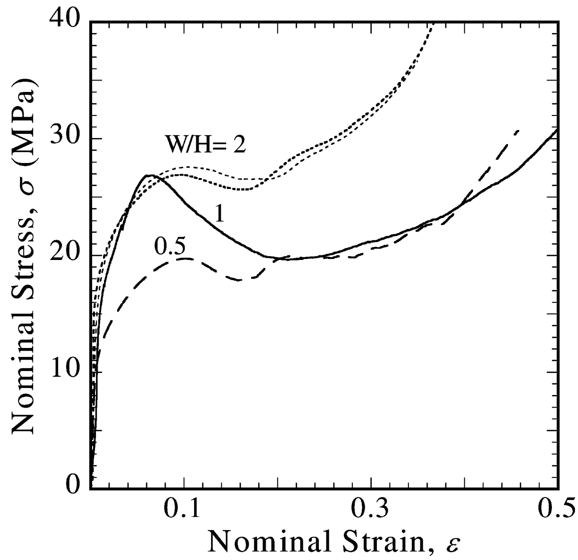

Fig. 8. The effect of specimen width to height ratio W/H upon the compressive stress versus strain response of woven core sandwich plates of relative density $\bar{\rho} = 0.23$, aspect ratio $L/H = 4$ and height to cell size ratio $H/d = 5$.

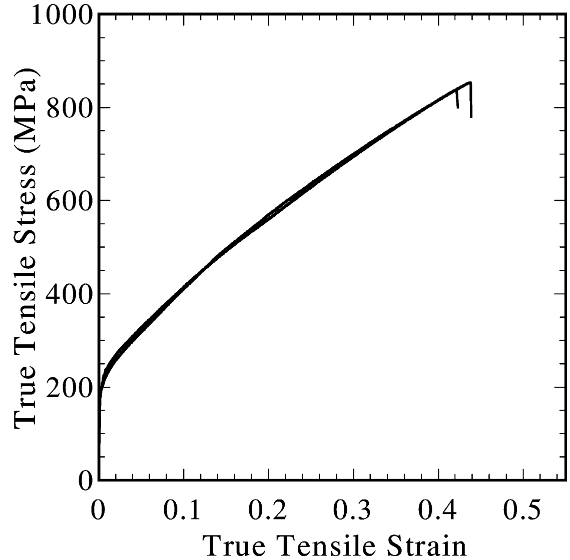

Fig. 9. Measured tensile stress versus strain response of the as-brazed 316 stainless steel.

6. Comparison of predicted and measured responses

6.1. Elastic properties of the woven sandwich core

A comparison of the predicted modulus (2.4) and the measured unloading modulus is shown in Fig. 10(a) as a function of the aspect ratio L/H for the three relative densities of cores tested. The measured and predicted values are in reasonable agreement and both show that the modulus decreases sharply with decreasing L/H for aspect ratios $L/H < 4$. The analysis assumes that the stiffness of the woven core is governed by axial stretching of the wire elements and consequently the predicted modulus scales linearly with $\bar{\rho}$; this scaling relation was confirmed by the measurements.

6.2. Plastic properties of the woven sandwich core

The strong strain hardening behavior of 316 stainless steel suggests that the peak stress of the woven core material is governed by plastic buckling of the wire segments in the mode sketched in Fig. 3. A photograph of a representative as-tested specimen ($\bar{\rho} = 0.23$, $L/H = 4$, $H/d = 5$ and $W/H = 1$), deformed to a uniaxial compressive strain of $\varepsilon \approx 11\%$ and sectioned along its mid-plane, is shown in Fig. 11. The image reveals that the buckling mode is similar to that sketched in Fig. 3, with each strut of the diamond-like unit cell buckling as a pin-ended strut. We proceed to compare the plastic buckling prediction (2.6) for this buckling mode (that is, $k = 1$) with the measured peak strengths. The bifurcation stress σ_c for the three relative densities of woven core makes use of the tensile stress versus strain response of the 316 stainless steel as given in Fig. 9. The values of bifurcation stress σ_c are calculated to be 274 MPa, 265 MPa and 259 MPa for $\bar{\rho} = 0.23$, 0.20 and 0.18, respectively.

A comparison of the predicted and measured peak strengths of the woven core is given in Fig. 10(b) as a function of the aspect ratio L/H. The peak strengths have been normalized by the measured yield strength Y of the as-brazed wire mesh material and the relative density $\bar{\rho}$ of the woven core. The predicted values of the peak strength are slightly below the measurements; imperfections would produce the opposite effect. This can be rationalized by noting that the buckling analysis is accurate for long slender columns with length to radius ratios $\ell/a > 10$. However, the high relative densities woven cores tested in this study have stubby struts with aspect ratios $\ell/a < 8$: these stubby struts are more resistant to buckling than predicted by the simple analysis employed here.

6.3. Comparison with other core topologies

The performance of stainless steel wove cores is compared with that for alternative cores in Fig. 12 which includes data for AL6XN stainless steel pyramidal cores (Zupan and Fleck, 2004), aluminum egg-boxes (Zupan et al., 2003), 304 stainless-steel square honeycombs (Cote et al., 2004a, 2004b) and aluminum alloy metal foams (Ashby et al., 2000). For a broader review of core topologies, see Wadley et al. (2003).

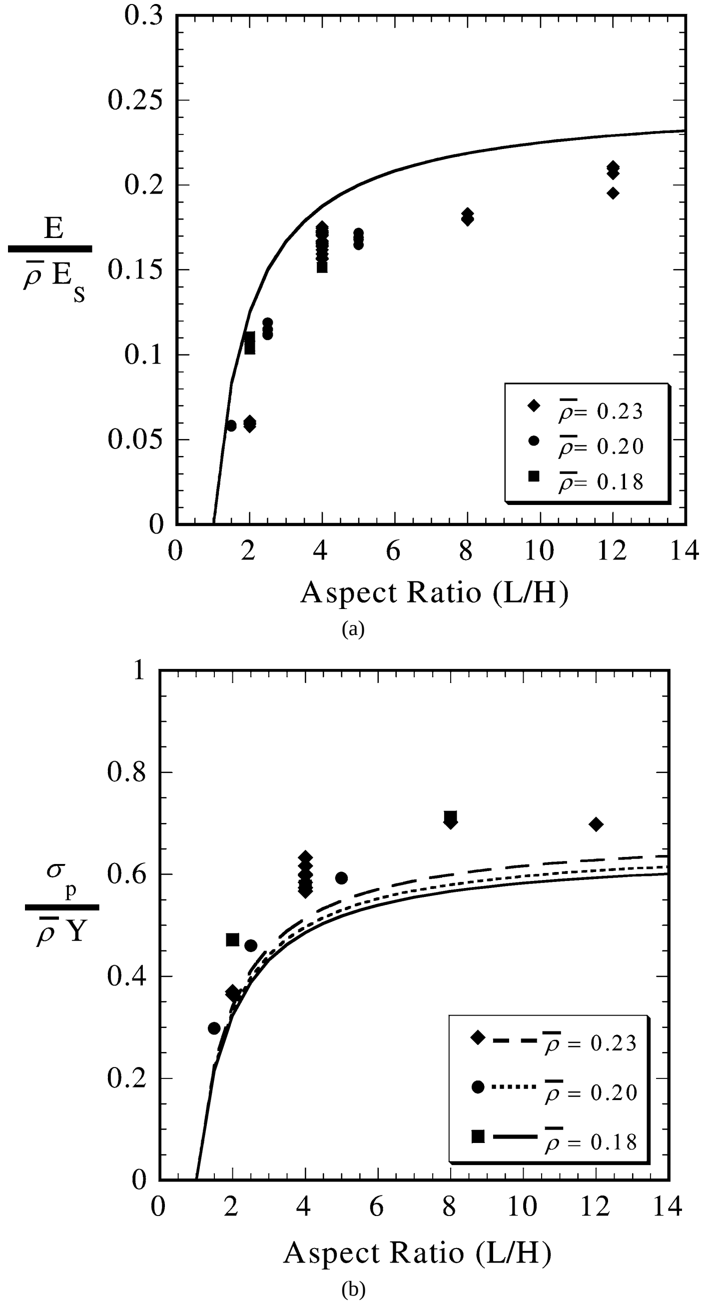

Fig. 10. Comparison of the measured and predicted (a) Young's modulus E and (b) peak collapse strength σ_P as a function of the aspect ratio of the sandwich core L/H.

The woven core is seen to out-perform the metal foams and egg-box materials but is weaker than the pyramidal and square-honeycombs for the same relative density.

7. Concluding remarks

Metal textile cores of sandwich plates have been manufactured by brazing together grade 316 stainless steel wire mesh. Analytical formulae for the out-of-plane modulus and compressive yield strength indicate that the stiffness and strength of the cores scales linearly with relative density $\bar{\rho}$: the nodal connectivity is sufficiently large for the out-of-plane properties to

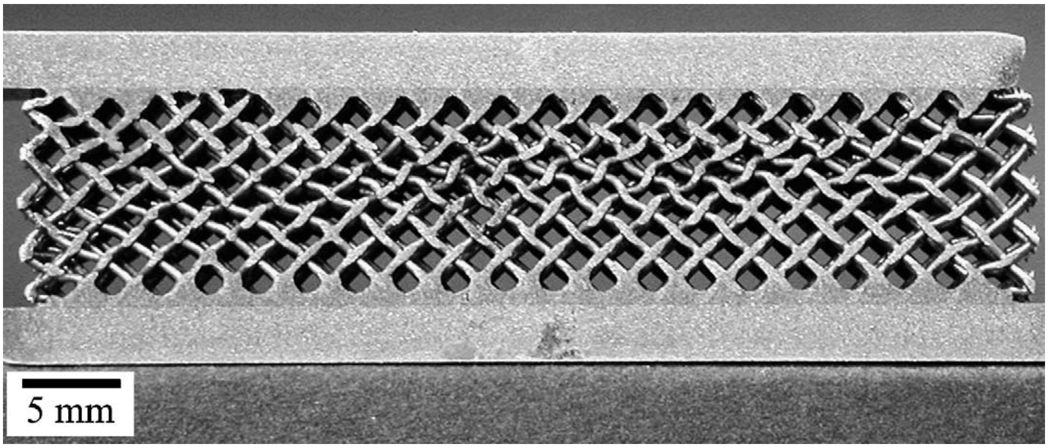

Fig. 11. Photograph of a woven core specimen compressed to just beyond the peak strength, $\varepsilon \approx 11\%$, and sectioned along its mid-plane. The photograph shows a buckling mode similar to that sketched in Fig. 3. $\bar{\rho} = 0.23$, $L/H = 4$, $H/d = 5$ and $W/H = 1$.

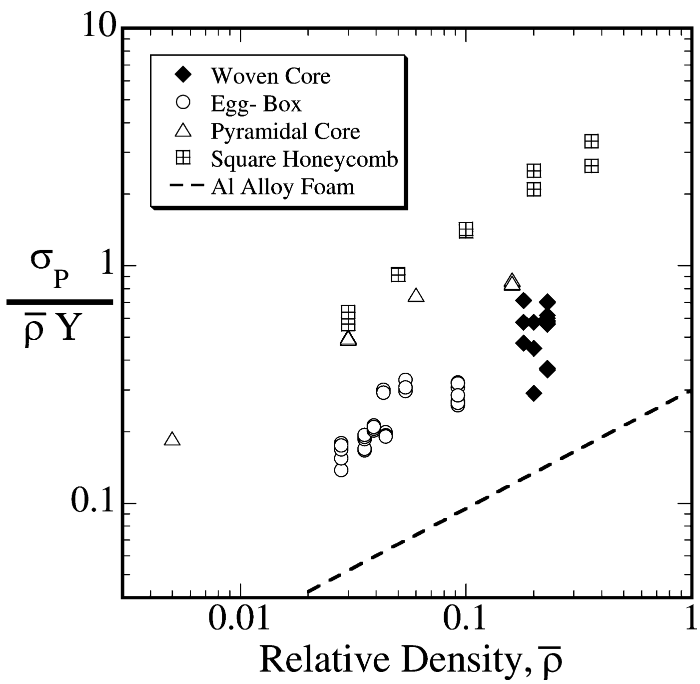

Fig. 12. Comparison of normalized core strength as a function of relative density for competing sandwich panel core topologies.

be governed by stretching rather than bending of the wire struts of the core. The peak strength does not scale precisely with relative density, as the wire struts become more stocky and possess an increased plastic bifurcation strength with increasing $\bar{\rho}$. The woven cores exceed 90% of their maximum achievable stiffness and strength for plates with aspect ratio $L/H > 8$ and core width to height ratio $W/H > 1$, see Figs. 10 and 8, respectively.

It is concluded that the out-of-plane properties of woven cores compare favorably with those of cellular materials such as metal foams, Ashby et al. (2000), but is the strength to weight ratio of the woven cores is lower than the pyramidal and square-honeycomb core topologies.

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