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Material selection in sandwich beam construction

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Abstract

A systematic procedure is presented for comparing the relative performance of sandwich beams with various combinations of materials in three-point bending. Operative failure mechanisms are identified and failure maps are constructed. The geometry of sandwich beams is optimised to minimise the mass for a required load bearing capacity in three-point bending.

Keywords: Sandwich materials; Analytical solutions; Optimisation; Beam; Bending

1. Introduction

Sandwich structures, widely used in aerospace and naval applications, tend to be limited to a small range of material combinations. For example, a metallic foam core is generally combined with a metal face sheet; a composite face is usually coupled with a polymeric foam core or a resin-impregnated paper honeycomb. Ashby and Bréchet [2] demonstrate that better performance may be achieved by using *hybrid* sandwich beams comprising non-traditional pairs of materials. This study presents a systematic method for evaluating novel material combinations for sandwich beams in three-point bending with an emphasis on minimum-weight design.

The purpose of optimal design for weight is to select the sandwich beam with the least mass for a given load-carrying capacity. Here, sandwich beams loaded in three-point bending are used as exemplary, and the following treatment deals exclusively with sandwich beams loaded as shown in Fig. 1. This study follows that of Gibson and Ashby [5] in their analysis of sandwich beam stiffness and production of failure mechanism maps, and draws upon extensive work on sandwich beam strength as reviewed by Zenkert [8].

2. Construction of a collapse mechanism map

Consider a simply supported sandwich beam loaded in three-point bending as sketched in Fig. 1. Let L be the beam length between the supports, b the width of the beam, c the core thickness, and t_f the face thickness. The relevant material properties for the core are the Young's modulus E_c, shear modulus G_c, compressive strength σ_c, and shear strength τ_c; for the face sheets, the pertinent properties are the compressive strength σ_f and Young's modulus E_f. The transverse mid-point deflection is δ due to an applied transverse load P.

It is well known that sandwich beams fail by one of several competing failure modes [5]; the operative mode is determined by the beam geometry, material properties, and the loading configuration. Four failure modes which regularly arise in sandwich beams in three-point bending are core shear, face yield or microbuckling, ductile indentation, and elastic indentation.

2.1. Competing collapse modes

Core shear failure occurs when the shear strength of the core is exceeded, and the peak strength P_{CS} is predicted by

$$P_{CS} = 2b(t + c)\tau_c. \tag{1}$$

Face yield or microbuckling occurs when the axial stress in the sandwich face attains the yield or microbuckling strength of the face material. Both failure modes are predicted by the expression:

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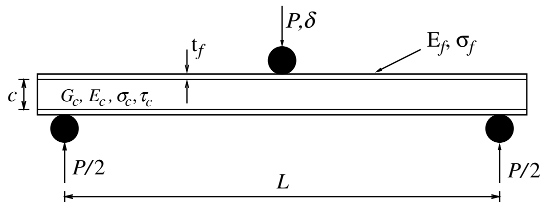

Fig. 1. Geometry of sandwich beams.

$$P_{\rm M} = \frac{4b(t+c)\sigma_{\rm f}}{L}. \tag{2}$$

Expressions (1) and (2) are provided in Zenkert [8] or Allen [1]. Two modes of indentation are used here. The first, described by Ashby et al. [3], is relevant to sandwich beams with metal foam cores and ductile faces. The faces are assumed to form plastic hinges at the boundaries of the indentation region. The expression for peak load in *ductile indentation* is

$$P_{\rm ID} = 2bt(\sigma_{\rm c}\sigma_{\rm f})^{1/2}. \tag{3}$$

An alternative indentation mode, modelled by Steeves and Fleck [6], is relevant to sandwich beams for which the indenting face sheet remains elastic while the core yields plastically. The faces behave as a beam column upon a non-linear foundation (provided by the core) and the peak indentation load is

$$P_{\rm IE} = bt\left(\frac{\pi^2(t+c)E_{\rm f}\sigma_{\rm c}^2}{3L}\right)^{1/3}. \tag{4}$$

This failure mode will be referred to as *elastic indentation*.

It is convenient to express the geometrical and material parameters in the non-dimensional form:

$$\bar{t} = t/c; \quad \bar{c} = c/L; \quad \bar{\sigma} = \sigma_{\rm c}/\sigma_{\rm f};$$
$$\bar{\tau} = \tau_{\rm c}/\sigma_{\rm f}; \quad \overline{E} = E_{\rm f}/\sigma_{\rm f}; \quad \text{and} \quad \bar{\rho} = \rho_{\rm c}/\rho_{\rm f}. \tag{5}$$

Now define a load index $\widehat{P}$ as

$$\widehat{P} = \frac{P}{bL\sigma_{\rm f}}. \tag{6}$$

The mass M of a sandwich beam is given by

$$M = bL(2t\rho_{\rm f} + c\rho_{\rm c}) \tag{7}$$

and the non-dimensional mass index $\widehat{M}$ is

$$\widehat{M} = \frac{M}{bL^2\rho_{\rm f}} = \bar{c}(2\bar{t} + \bar{\rho}), \tag{8}$$

where the non-dimensional parameters from (5) are substituted into (7) and (8).

Similarly, the failure loads (1)–(4) can be non-dimensionalised for core shear failure as

$$\widehat{P}_{\rm CS} = 2\bar{\tau}(\bar{t}+1)\bar{c} \tag{9}$$

for microbuckling or face yield as

$$\widehat{P}_{\rm M} = 4\bar{t}(\bar{t}+1)\bar{c}^2 \tag{10}$$

for ductile indentation as

$$\widehat{P}_{\rm ID} = 2\bar{t}\bar{c}\bar{\sigma}^{1/2} \tag{11}$$

and for elastic indentation as

$$\widehat{P}_{\rm IE} = \left(\frac{\pi^2\bar{\sigma}^2\overline{E}}{3}\right)^{1/3}\bar{t}(\bar{t}+1)^{1/3}\bar{c}^{4/3}. \tag{12}$$

2.2. Collapse mechanism maps

Gibson and Ashby [5] developed methods for the graphic display of the collapse regimes for competing failure mechanisms. Such mechanism maps are useful tools for the optimisation of sandwich beams. The optimisation strategy employed here is to find the combinations of $\bar{t}$ and $\bar{c}$ to minimise the mass index $\widehat{M}$ for a given material combination and structural load index $\widehat{P}$. The procedure is as follows:

(i) Calculate the weakest and therefore the active collapse mode. This gives the regimes of dominance of each collapse mode in $(\bar{c}, \bar{t})$ space, and thereby generates the collapse mechanism map.
(ii) Seek the value of $(\bar{c}, \bar{t})$ that minimises $\widehat{M}$ for any assumed value of $\widehat{P}$.
(iii) Repeat step (ii) for other values of $\widehat{P}$, in order to construct the minimum mass trajectory.

The trajectory of optimal design usually lies along the boundary between collapse mechanisms, but can also occur within the face yield/microbuckling domain, or within the elastic indentation domain. Fig. 2 is an

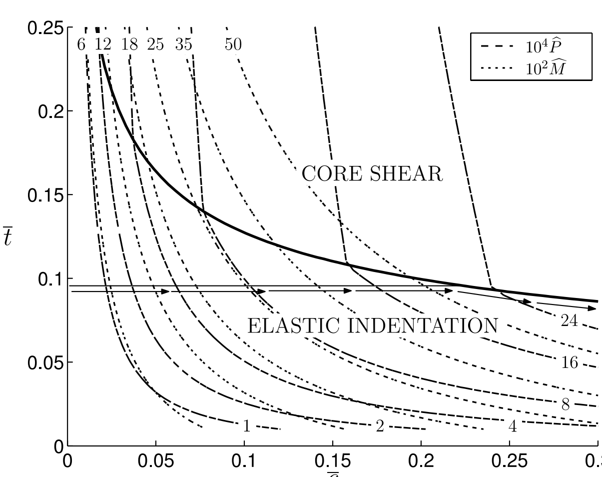

Fig. 2. Failure mechanism map for GFRP face sheets and medium density H100 foam core. Contours of mass index $10^2\widehat{M}$ and structural load index $10^4\widehat{P}$ are labelled with their numerical values. The solid bold line denotes the boundary between the indentation and core shear regimes, and the arrows follow the minimum weight trajectory.

Table 1
Material properties of representative constituents of sandwich beams

Material	Density (kg/m^3)	Tensile/compressive strength (MPa)	Shear strength (MPa)	Young's modulus (GPa)
CFRP faces	1600	650		65
GFRP faces	1770	350		30
Medium strength steel faces	8000	400		210
Steel square honeycomb core	800	20	20	
PVC foam core	100	1.45	1.66	

GFRP = glass fibre reinforced plastic; CFRP = carbon fibre reinforced plastic.

example of a failure mechanism map for a Hexcel Fibredux woven GFRP face and Divinycell H100 PVC foam core. (Properties for all materials are contained in Table 1.) Mass is minimised in $(\bar{c}, \bar{t})$ space along the trajectory where the gradient of the mass index is locally parallel to the gradient of the load index, or on a boundary between two regimes. In this example, only the failure modes of elastic indentation and core shear are active. Contours of structural load index $\widehat{P}_{\min}$ and mass index $\widehat{M}$ are superimposed on the map, and a trajectory of minimum weight design is along the path where the contours of $\widehat{M}$ are parallel to the contours of $\widehat{P}_{\min}$. The minimum weight design trajectory begins in the elastic indentation region for small values of $\bar{c}$ and switches to the boundary between elastic indentation and core shear at $\bar{c} \approx 0.22$.

A similar map for a sandwich beam with a steel face and a steel core comprising a square honeycomb is displayed in Fig. 3. Here, the three operative failure modes are face yield, ductile indentation, and core shear. The trajectory of minimum weight design starts in the face yield region, and passes along the face yield—ductile indentation boundary and then along the core shear—ductile indentation boundary as $\widehat{P}$ increases.

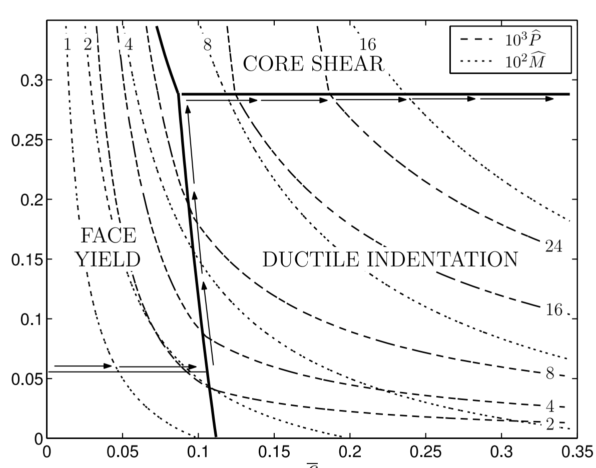

Fig. 3. Failure mechanism map for steel face sheets and square steel tube core. Contours of mass index $10^2\widehat{M}$ and structural load index $10^3\widehat{P}$ are labelled with their numerical values. Solid bold lines indicate the boundaries between failure mechanisms regimes, and arrows follow the minimum weight trajectory.

2.3. Minimum weight design as a function of structural load index

Expressions for minimum weight design within each failure regime and along the regime boundaries are now obtained. In the microbuckling region, we have

$$\widehat{M}_{\min} = (\bar{\rho}(2-\bar{\rho})\widehat{P})^{1/2}, \tag{13}$$

with the optimal value of $\bar{t}$ given by

$$\bar{t} = \frac{\bar{\rho}}{2(1-\bar{\rho})}. \tag{14}$$

Similarly, within the elastic indentation region, the minimum mass index $\widehat{M}_{\min}$ is

$$\widehat{M}_{\min} = 4\left(\frac{\bar{\rho}(2-\bar{\rho})^3}{9\pi^2\bar{\sigma}^2\overline{E}}\right)^{1/4}\widehat{P}^{3/4}, \tag{15}$$

with $\bar{t}$ given by

$$\bar{t} = \frac{3\bar{\rho}}{2(1-2\bar{\rho})}. \tag{16}$$

For the boundary between elastic indentation and face yield/microbuckling, the minimum mass index $\widehat{M}_{\min}$ is

$$\widehat{M}_{\min} = \frac{\pi\bar{\rho}\bar{\sigma}}{8}\left(\frac{\overline{E}}{3}\right)^{1/2} + \frac{2(2-\bar{\rho})}{\pi\bar{\sigma}}\left(\frac{3}{\overline{E}}\right)^{1/2}\widehat{P}. \tag{17}$$

On the boundary between elastic indentation and core shear, it is found that

$$\widehat{M}_{\min} = \left(\frac{6\bar{\tau}}{\pi^2\bar{\sigma}^2\overline{E}}\right)^{1/3}(2-\bar{\rho})\widehat{P}^{2/3} + \frac{\bar{\rho}\widehat{P}}{2\bar{\tau}}. \tag{18}$$

Similarly, on the elastic indentation–ductile indentation boundary, we have

$$\widehat{M}_{\min} = \frac{24\bar{\rho}}{\pi^2\bar{\sigma}^{1/2}\overline{E}} - \widehat{P}\left(\frac{\bar{\rho}-2}{2\bar{\sigma}^{1/2}}\right). \tag{19}$$

The remaining expressions are, for the face yield/microbuckling–core shear boundary:

$$\widehat{M}_{\min} = \frac{\widehat{P}\bar{\rho}}{2\bar{\tau}} + \frac{\bar{\tau}}{2}(2-\bar{\rho}) \tag{20}$$

for the boundary between face yield/microbuckling and ductile indentation:

$$\widehat{M}_{\min} = \frac{\bar{\rho}}{2}\left(\bar{\sigma}^{1/2} + \frac{\widehat{P}(2-\bar{\rho})}{\bar{\rho}\bar{\sigma}^{1/2}}\right) \tag{21}$$

and for the boundary between core shear and ductile indentation:

$$\widehat{M}_{\min} = \frac{\widehat{P}}{2}\left(\frac{\bar{\rho}}{\bar{\tau}} + \frac{2-\bar{\rho}}{\bar{\sigma}^{1/2}}\right). \tag{22}$$

To assess the performance of any combination of materials, the explicit relationship between the structural load index $\widehat{P}$ and the minimum mass index $\widehat{M}_{\min}$ is required. The definition (6) for $\widehat{P}$ and (8) for $\widehat{M}$ involve the strength σ_f and density ρ_f of the face sheets. This makes it difficult to explore the effect of the choice of face sheet upon sandwich beam performance. To allow for a direct comparison of the performance of various material combinations, the normalised values $\widehat{P}^N$ of $\widehat{P}$ and $\widehat{M}^N$ of $\widehat{M}$ are introduced, by using the strength σ_s and density ρ_s of a medium strength steel (as listed in Table 1):

$$\widehat{P}^N \equiv \frac{\sigma_f}{\sigma_s}\widehat{P} \tag{23}$$

and

$$\widehat{M}^N \equiv \frac{\rho_f}{\rho_s}\widehat{M}. \tag{24}$$

This procedure minimises mass for a given strength requirement. Other optimisations could be performed; for example, mass could be minimised for a given structural stiffness. Performance criteria, such as corrosion resistance, fracture toughness, or fatigue resistance, can be treated similarly.

The minimum weight designs for selected sandwich beams are presented in Fig. 4, upon making use of the material property data listed in Table 1. These comprise the three face sheet materials carbon fibre reinforced plastic (CFRP), glass fibre reinforced plastic (GFRP) and steel plate; two core materials are addressed: Divinycell H100 PVC foam and a steel core composed of a square honeycomb (see [4]). (The properties of steel lattice core are very close to those of the H100 PVC foam core, for the case of a pyramidal core of density 100 kg/m^3; see [7].) To select the material combination for minimum weight design, one chooses the materials which minimise $\widehat{M}_{\min}$ for any selected value of $\widehat{P}$. It is clear from Fig. 4 that, at low values of $\widehat{P}$, the optimal choice of materials is CFRP faces with a PVC foam core. For higher values of $\widehat{P}$, CFRP faces with a steel square honeycomb core is preferred. Here, we have used only five exemplary materials; a full optimisation would involve a database including a much larger selection of materials. Note also that not all combinations of materials are compatible (for manufacturing or other reasons) and that some engineering judgement is required to assess the viability of material combinations.

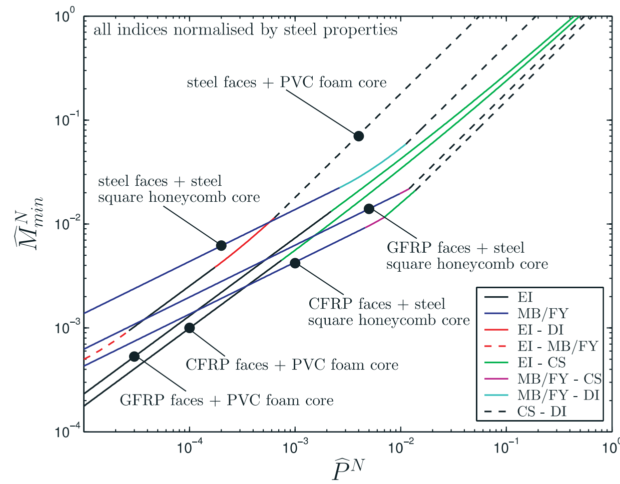

Fig. 4. Variation of minimum mass index $\widehat{M}^N_{\min}$ with structural load index $\widehat{P}^N$ for a variety of material combinations. Minimum mass is found by choosing the material system with the minimum $\widehat{M}_{\min}$ for a given $\widehat{P}$. EI = elastic indentation; MB/FY = microbuckling/face yield; CS = core shear; DI = ductile indentation.

3. Concluding remarks

The above analysis demonstrates a systematic method for choosing the best materials for achieving minimum mass design of a sandwich beam under given loading conditions. The set of materials considered is intended to be illustrative but not exhaustive. It does, however, illustrate the potential for using non-traditional material combinations in sandwich construction: while a 'traditional' combination of carbon-fibre composite face sheet and polymer foam core is optimal for low structural load indices, at higher structural load indices the hybrid carbon-fibre composite—steel square honeycomb core beam is optimal. Recently developed lattice cores [7] have similar or superior properties to polymeric foam cores, and opportunities remain to develop new core topologies and new material combinations to maximise sandwich beam performance over the entire range of structural load index.

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