Fracture of Brittle Lattice Materials: A Review

Ignacio Quintana-Alonso and Norman A. Fleck

Cambridge University Engineering Department, Trumpington St, Cambridge CB2 1PZ, UK

E-mail addresses: iq201@cam.ac.uk and naf1@cam.ac.uk

Abstract The mechanics of failure for elastic-brittle lattice materials is reviewed. Closed-form expressions are summarized for fracture toughness as a function of relative density for a wide range of periodic lattices. A variety of theoretical and numerical approaches has been developed in the literature and in the main the predictions coincide for any given topology. However, there are discrepancies and the underlying reasons for these are highlighted. The role of imperfections at the cell wall level can be accounted for by Weibull analysis. Nevertheless, defects can also arise on the meso-scale in the form of misplaced joints, wavy cell walls and randomly distributed missing cell walls. These degrade the macroscopic fracture toughness of the lattice.

1. Introduction

Lattice materials are enjoying increasing use in engineering applications such as the core of sandwich panels. Extensive research has been conducted into the prediction of stiffness and strength for a wide range of 2D and 3D lattices. For example, the in-plane elastic properties of various lattice topologies are now well understood [1,2,3]. In contrast, little work has been done on their fracture properties and damage tolerance.

The response of lattice materials to several types of defects has been investigated by numerical, analytical and experimental methods. For instance, the mechanical properties of regular hexagonal lattices with defects consisting of missing cells were analysed by Guo & Gibson [4] using the finite element (FE) method. The effect of holes and rigid inclusions on the elastic modulus and yield strength are studied in Chen et al. [5]. Defects in the form of randomly fractured cell-walls have been examined numerically [6] and experimentally [7]. The influence of defect size and cell size on the tensile strength of notched lattices was studied in Andrews & Gibson [8] by means of FE simulations. Chen et al. [9] provide a comprehensive study of this wide range of geometrical imperfections. They found that fractured cell-walls produce the largest knock-down effect on the yield strength of hexagonal lattices.

Damage tolerance is a broad subject, as illustrated by the variety of defects examined in the above studies. The purpose of this review is not to provide an exhaustive list. Rather, we focus on the response of 2D brittle lattices to the presence of a crack or sharp notch. The dependence of fracture toughness upon microstructure is reviewed and the applicability of conventional fracture mechanics concepts to the discrete lattice is assessed.

1.1 Fracture Mechanics Concepts

For a linear-elastic material, the *stress intensity factor* K defines the magnitude of the dominant component of the local stresses near the crack tip. It accounts for the geometry of the body containing the crack and the type of loading to which it is subjected. The power and utility of K is that this macroscopic parameter overcomes our lack of knowledge of the microscopic fracture events at the crack tip.

Rapid, unstable crack advance will occur when the local stresses reach some critical value, which will be constant for a given material. It immediately follows that these critical local conditions will correspond to a critical value of the stress intensity factor, $K = K_C$, which we can easily quantify and then use as a measure of a material's resistance to rapid crack advance; we call this resistance the material's *fracture toughness*, K_C.[1]

There are three types of loading that a crack can experience. Mode I loading, where the principal load is applied normal to the crack plane, tends to open the crack. Mode II corresponds to in-plane shear loading and tends to slide one crack face with respect to the other. Mode III refers to out-of-plane shear. A cracked body can be loaded in any one of these modes, or a combination of two or three modes giving rise to mixed-mode fracture. Mode I is technically the most important; the discussions in this review are limited to modes I and II.

Conventional linear elastic fracture mechanics (LEFM) was developed for a continuous medium. Continuum theory can predict the stresses near the crack tip, but it is the material's microstructure that determines the critical conditions for fracture. It is well known that the length scales associated with the microscopic events that lead to failure of the material at the crack tip must be sufficiently small

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[1] It is worth emphasising the physical origin of each side of the fracture criterion $K = K_C$. The stress intensity factor on the left hand side is a mechanics parameter, its value determined by the geometry, stress level and crack length in the component, and is a measure of the cracking effort being applied to the component; whilst the right hand side is a material constant, a measure of the particular material's ability to resist rapid crack advance.

compared with the size of the K-field. In a lattice material, this means that for K to define uniquely the crack-tip stress conditions and be a valid failure criterion, the cell size must be small compared to the size of the singularity zone governed by the K-field. Otherwise, the continuum assumption is invalid and linear elastic fracture mechanics (LEFM) becomes inapplicable.

1.2 Outline of this Review

This review is arranged thematically rather than chronologically. It is organised according to the different methods and models used to determine the fracture properties of lattices. First, analytical estimates of fracture toughness obtained by classical beam theory are presented. These studies are based on the assumption that the cell-walls are adequately modelled by a lattice composed of rigidly connected Euler beams. Second, we consider generalised continuum theories that apply homogenisation techniques to the lattice. Third, major contributions obtained by the finite element method are reviewed in detail. We then briefly mention atomic lattice models. Next, the main results obtained by the representative cell method are summarised. Finally, we point out the few experimental studies dealing with the determination of the fracture toughness of lattices.

2. Classical Beam Theory

Ashby [10] and Maiti et al. [11] is the starting point for all subsequent investigations of the fracture toughness of lattice materials. The concept of effective fracture toughness K_C is based upon the existence of a K-field on a length scale much larger than the cell size of the lattice. Ashby [10] made use of LEFM concepts to estimate the fracture toughness of a hexagonal lattice (Fig. 1a). The stress field of an equivalent linear-elastic continuum was used to calculate the stresses on the discrete cell walls of the lattice directly ahead of the crack tip. The macroscopic fracture toughness was then estimated by assuming that the critical strut directly ahead of the crack tip fails when the maximum tensile stress within it attains the tensile fracture strength σ_{TS} of the parent solid material[2].

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[2] Tensile fracture strength σ_{TS} and modulus of rupture σ_f are used indistinctively in this review. The modulus of rupture σ_f is the maximum surface stress in a bent beam at the instant at which it fractures. If the beam is made of a brittle solid, like the cell wall discussed here, the fracture initiates at a microcrack (usu-

2.1 The Hexagonal Lattice

These analytical models predict that the mode I fracture toughness K_{IC} of the regular hexagonal lattice scales linearly with σ_{TS}, quadratically with stockiness t/l, and with the square root of the cell size l as

$$K_{IC} = 0.53\left(t/l\right)^2 \sigma_{TS}\sqrt{l} \tag{1}$$

This approach is described in detail in the fundamental monograph by Gibson & Ashby [1].

In-plane shear stress is the primary loading in sandwich panels, producing a mode II or mixed-mode fracture. The mixed-mode fracture criteria for solid materials are invalid for lattice materials because failure mechanisms are different due to the discreteness of the lattice. Accordingly, Huang & Lin [13] analysed the mixed-mode fracture for hexagonal lattices under a combined loading of uniform tensile and in-plane shear stresses. As a first step they derived the expression for mode II fracture toughness K_{IIC} using dimensional arguments in cooperation with the near tip singular stress field of a continuum. It was concluded that the mode I and II fracture toughness have the same dependence on cell size, stockiness and fracture strength exhibited by Eq. (1), with the coefficient to be determined empirically or numerically.

Huang & Lin [13] considered only cell-wall bending (acceptable for the hexagonal topology) to obtain a mixed-mode fracture criterion which is a linear combination of K_I/K_{IC} and K_{II}/K_{IIC}. Fleck & Qiu [14] showed that it is in fact piece-wise linear. The mixed-mode fracture criterion for brittle lattice materials contrasts with that of solid materials, for which usually a smooth convex envelope can be described by an ellipse. Huang & Lin [13] compared the theoretical mixed-mode fracture criterion with limited experimental data for PVC foams, claiming agreement between theory and experiment. Nevertheless, their conclusion should be regarded with caution, given the scarcity and scatter of the data.

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ally a surface microcrack) in the wall and propagates catastrophically. On average, the modulus of rupture is a little larger than the tensile strength σ_{TS} because, in bending, only one surface of the beam sees the maximum tensile stress; in simple tension the entire beam is stressed uniformly, so a given microcrack is less likely to be stressed in bending than in simple tension. The statistics of the problem (see, for example, Davidge [12]) show that the modulus of rupture is typically about 1.1 times larger than the tensile strength.

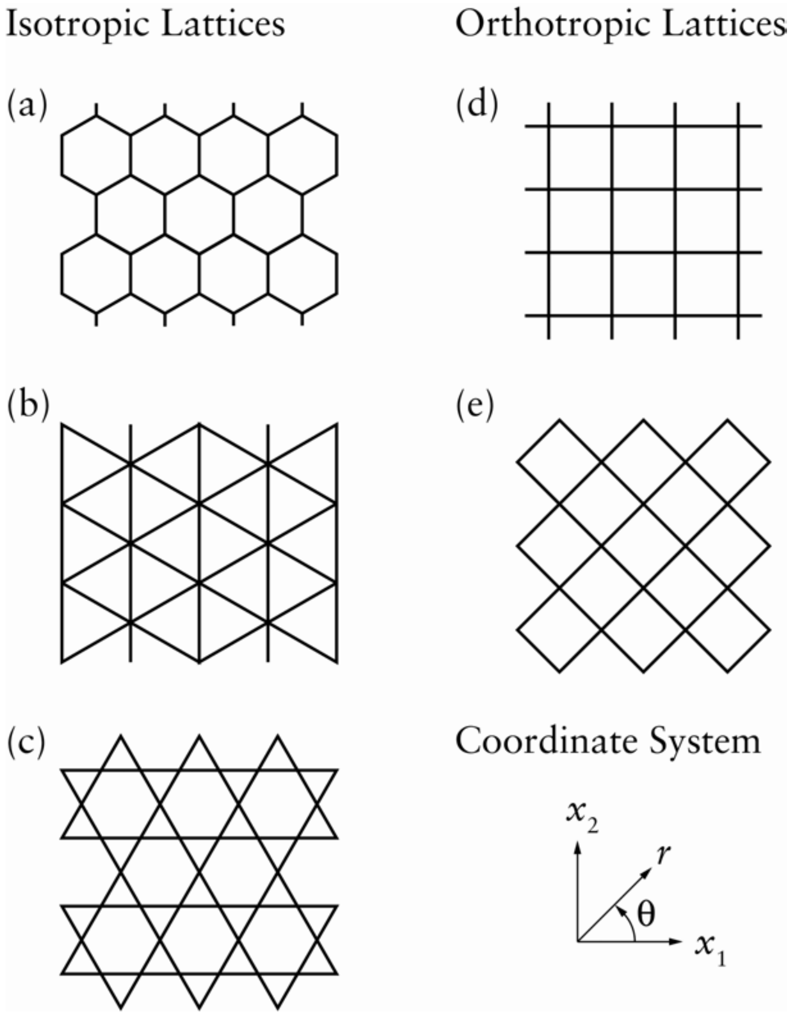

Fig. 1. Planar lattice topologies: (a) hexagonal, (b) triangular, (c) Kagome, (d) square, and (e) diamond-celled.

2.2 Other 2D Lattices

Fleck & Qiu [14] have evaluated K_{IC} and K_{IIC} for the isotropic triangular and Kagome lattices (Fig. 1b, c). Similar expressions to Eq. (1) are derived. They find that K_{IC} scales linearly with stockiness t/l for the triangular lattice, and scales as $(t/l)^{1/2}$ for the Kagome topology. More recently, Quintana-Alonso & Fleck [15] have considered the fracture toughness of the orthotropic, diamond-celled lattice, see Fig. 1e. Their model is in the spirit of the analyses of Gibson & Ashby [1] for the hexagonal lattice, and of Fleck & Qiu [14] for the triangular and Kagome lattices. Quintana-Alonso & Fleck [15] find that K_{IC} scales linearly with t/l for the diamond-celled topology.

2.3 Statistics of Brittle Failure

The first studies on the fracture properties of brittle lattice materials assumed that the tensile strength of a single cell wall is constant. In practice this is unrealistic. Brittle fracture is a stochastic process, dependent on the presence and distribution of defects in the struts. Depending on the width of this distribution, the failure of the first strut may or may not trigger the failure of the whole. The tensile and compressive strengths of ceramic lattice materials are significantly influenced by the flaw size distribution within them. As a result, variability in the strength of brittle lattice materials is expected.

Weibull [16] proposed an empirical formulation with a simple statistic distribution to describe the strength variability in brittle materials such as concrete, wood and glass fibre. The Weibull modulus m describes the flaw size distribution such that the material becomes less brittle as m increases. Huang & Gibson [17] have made use of Weibull statistics to include probabilistic effects in the fracture toughness of hexagonal and square lattices. They concluded that the fracture toughness increases with increasing cell size for a Weibull modulus $m > 4$, is insensitive to cell size for $m = 4$, and decreases with increasing cell size for $m < 4$. This result was later used by Huang & Chou [18] to modify the model for failure envelopes of lattice materials under in-plane biaxial loading [19].

The validity of Huang & Gibson's [17] result is limited by the assumption that failure always occurs near the crack tip and, thus, only the critical strut directly ahead of the crack tip needs to be considered. This assumption holds true for large values of Weibull modulus m. But for small m, the variability in strength may be sufficiently high for struts remote from the crack tip to fail. Quintana-Alonso and Fleck [20] prove that for $m < 4$, struts remote from the crack tip fail: the effective stressed volume becomes sufficiently large for the predicted fracture toughness to drop to zero according to Weibull theory.

3. Generalised Continuum Theories

An alternative analytical method for fracture toughness evaluation is presented by Chen et al. [21]. The authors determined the fracture toughness of lattices with hexagonal, triangular and square cells (Fig. 1a, b, d) by replacing the discrete lattice with a generalised continuum described within Cosserat theory[3]. The model is based on equating the continuum approximation of the strain energy of the discrete lattice to the strain energy of an equivalent micro-polar continuum. For the prototypical regular hexagonal lattice they obtained

$$K_{IC} \approx 1.8(t/l)\sigma_{TS}\sqrt{l} \tag{2}$$

This result gives a much higher toughness than Eq. (1), particularly for low-density materials, due to the linear rather than quadratic dependence upon t/l. The experimental data quoted by Gibson & Ashby [1] support the quadratic dependence of K_{IC} upon t/l. As indicated by Fleck & Qiu [14], the discrepancy can be traced to the fact that Chen et al. [21] assumed affine deformation in their calculation of the effective properties of the Cosserat medium: this gives too stiff a response as it neglects the dominant contribution of cell wall bending under uniform loading. The analysis of Maiti et al. [11] correctly includes the effect of cell wall bending and thereby gives the correct functional dependence of fracture toughness upon t/l, as given by Eq. (1).

It is interesting to note that the possibility of applying generalised continuum theories to lattice structures and frames was already discussed in the pioneering works of Banks & Sokolowski [22] and Bazant & Christensen [23], who developed an analogy between a micro-polar medium and an uncracked rectangular lattice. They showed that to obtain accurate results, the continuum approximation of potential energy of individual struts must be expressed exactly up to terms with second order derivatives of joint rotation. Specifically, this approach is necessary to derive the correct micro-polar constants relating the in-plane moments and bending curvature.

The continuum description of lattice materials enables the use of powerful analytical methods, such as asymptotic analysis, Fourier transform and Wiener-Hopf technique, to study the fracture of lattice materials. For instance, homogenisation models for cracks in lattice materials with triangular, square and hexagonal cells were studied by Antipov et al. [24,25] with emphasis on crack propagation. They

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[3] Classical continuum theory is well-suited for situations where the variations in stresses and strains are smooth. However, in many circumstances this is not necessarily the case (e.g. near crack tips or boundary layers) and one resorts to enhanced continuum theories (also called enriched or generalised). Such theories include information on the microstructure and take into account the non-uniformity of stresses and strains at the micro-scale. One of the simplest generalised theories is Cosserat (or micro-polar) theory, in which the interaction between neighbouring material points is governed by a moment vector in addition to the force vector from classical continuum theory.

derive explicit asymptotic formulae for the stress intensity factors, and in addition analyse interaction of cracks in the homogenised lattice. This procedure leads to similar results to those of Chen et al. [21]: for the hexagonal lattice the inferred fracture toughness is too large.

A significant body of research has evolved on generalised continuum modelling of 2D periodic lattices. This approach has an advantage over discrete modelling of the lattice especially in design selection exercises involving various cell topologies and complex domains. For example, various 2D lattices can be rank-ordered with respect to fracture toughness or notch resistance [26]. However, the generalised continuum approach has disadvantages: homogenisation techniques are unable to capture microscopic instabilities such as local buckling of individual struts in the discrete lattice.

4. Finite Element Modelling

4.1 Stress Analysis

Lattice materials can be viewed as rigid-jointed interconnected simple beams. Consequently, the methods of analysis developed for 2D beams and frames in structural engineering are used to analyse their stiffness and strength. One direct approach is to treat the material as a latticework and determine stresses in various members by a numerical method such as finite element (FE) analysis. This approach, in general, leads to the prediction of strength for a lattice material rather than its fracture toughness.

Within the framework of this numerical approach, the fracture toughness is calculated by the FE simulation of a finite lattice plate with several missing struts. Suppose the characteristic size of the plate is equal to L, the crack-like flaw modelled by contiguous removed struts has length $2a$, and the cell size is l, see Fig. 2. The requirement for the correct evaluation of the fracture toughness is expressed by the inequality

$$L \gg a \gg l \tag{3}$$

Huang & Gibson [17] used the FE model to examine a rectangular lattice plate subjected to uniaxial tension with a central finite length crack (Fig. 2). They found that for a mode I crack in a diamond-celled lattice the fracture toughness is approximately given by

$$K_{IC} = 1.7\left(t/l\right)^2 \sigma_{TS}\sqrt{l} \tag{4}$$

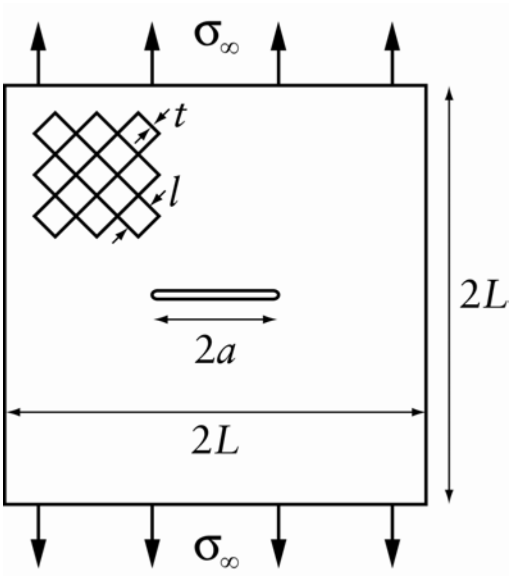

Fig. 2. Centre-cracked plate under uniaxial tension.

Previous attempts to model the fracture toughness of brittle lattices and foams applied well-known results for a continuum by assuming that the crack length is large relative to the cell size. In practice, this may be unrealistic, since short cracks of roughly three to four cells are common in brittle lattices and foams. Huang & Gibson [17] detected that Eq. (4) is valid when the stockiness t/l is lower than 0.2 and the ratio of crack length to cell size a/l is larger than 7. For lattices with $a/l < 7$, the effective fracture toughness is reduced by a factor which is insensitive to changes in the cell geometry. They attributed the discrepancy at ratios of t/l higher than 0.2 to the fact that axial stresses in the cell walls, neglected in the analytic model, become significant.

Quintana-Alonso & Fleck [15] demonstrate that the crack length required for the correct fracture toughness evaluation scales with the stockiness of the diamond-celled lattice as $a/l \propto (t/l)^{-2}$ and, consequently, exceeds the above value for small stockiness.

The K_{IC} formulations in LEFM are applicable for solid materials but have some restrictions on specimen geometry [27]. Similarly, the validity of the K_{IC} formulations and their corresponding specimen geometry restrictions must be assessed before they are employed to compute fracture toughness of lattice materials. The single-edge notched beam in three-point bend shown in Fig. 3 is the easy and common test for measuring K_{IC} of brittle lattices. Hence, Huang & Chiang [28] carried out a numerical examination of the restrictions on specimen geometry of the three-point bend test, including span S, height H and crack depth ratio a/B. These authors suggest that the fracture toughness measured from a three-point bend test is higher than that measured from a uniaxial test, and that the difference between the two measures is reduced as stockiness increases. This conclusion is inconsistent with the definition of fracture toughness – a material property independent of test geometry. Quintana-Alonso et al. [29] have recently reassessed the three-point bend problem and show that a *K*-dominated regime only exists for sufficiently large at/l^2. At low at/l^2 a crack-insensitive regime prevails.

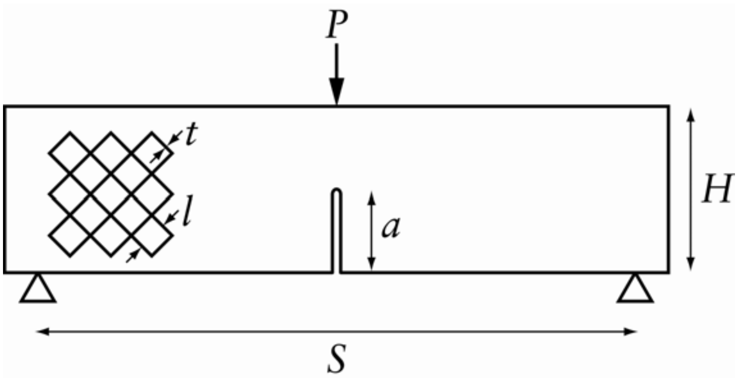

Fig. 3. Single-edge notched beam in three-point bending.

4.2 Boundary Layer Analysis

The numerical approach to the fracture toughness evaluation can be improved by the use of continuum fracture mechanics. A finite element problem is formulated for the finite lattice domain surrounding the tip of a macroscopic crack. A semi-infinite crack is introduced by breaking a series of discrete cell walls (see Fig. 4). The displacements u_i and rotations ψ of the beam ends are prescribed on the outer boundary according to the asymptotic *K*-field of a crack in an equivalent homogenous material possessing effective elastic properties. The fracture toughness of the lattice is calculated by relating the stresses and strains generated in the cell walls to the magnitude of the applied *K*-field. This formulation, referred to as a boundary layer analysis, was first employed by Schmidt & Fleck [30] to investigate crack growth initiation and subsequent propagation for regular and irregular elastic-plastic hexagonal lattices.

A seminal work on the fracture behaviour of 2D lattice materials is that of Fleck & Qiu [14]. These authors determined the fracture response of three isotropic lattices: hexagonal, triangular, and Kagome (Fig. 1). They found that the fracture toughness K_C of these lattices scales with stockiness t/l according to

$$\frac{K_C}{\sigma_{TS}\sqrt{l}} = D\left(\frac{t}{l}\right)^d \tag{5}$$

where the exponent d equals 2 for the hexagonal lattice, equals unity for the triangular lattice, and equals $1/2$ for the Kagome lattice. For each topology, the coefficient D is slightly less for mode II loading than for mode I loading, see Table 1.

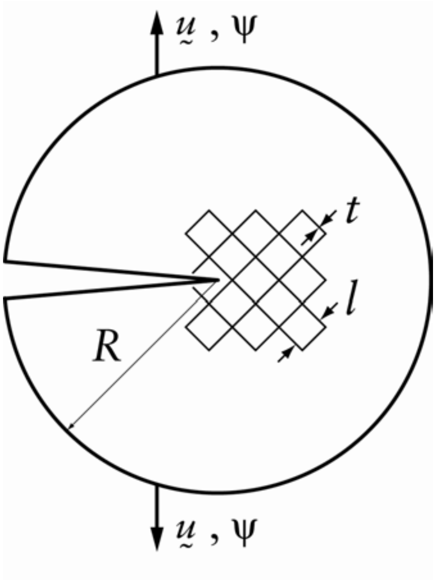

Fig. 4. Boundary layer analysis: lattice material with a macroscopic crack under *K*-control. The joint displacements and rotations associated to a *K*-field are prescribed on the outer boundary.

It is emphasised that the value of the exponent d has a dominant influence on the magnitude of the fracture toughness. For example, at a relative density of 10%, the fracture toughness of the Kagome lattice is 10 times greater than that of the hexagonal lattice (see Fig. 5). The unusually high fracture toughness of the Kagome lattice is attributed to the presence of an elastic zone of bending emanating from the crack tip into a remote stretching field. The Kagome lattice exhibits pronounced crack tip blunting due to the deformation band at the crack tip; this elastic blunting phenomenon reduces the stress levels at the crack tip and thereby increases the fracture toughness. Quintana-Alonso & Fleck [31] found a similar effect for the diamond-celled lattice. A particular feature of this lattice topology is a low resistance to shear along the $\pm 45^\circ$ directions; the elastic bending zones emanate from the crack tip along these directions.

Topology	Mode I		Mode II		Reference
	D	d	D	d	
Hexagonal	1.20	2	0.54	2	Fleck & Qiu [14]
	1.80	1	0.77	1	Chen et al. [21]
	0.53	2	-	-	Gibson & Ashby [1]
	0.43	2	-	-	Huang & Chiang [28]
Triangular	2.10	1	1.40	1	Fleck & Qiu [14]
	4.60	1	1.50	1	Chen et al. [21]
Kagome	0.27	1/2	0.15	1/2	Fleck & Qiu [14]
Square	0.56	1	0.34	3/2	Romijn & Fleck [36]
	0.71	1	0.20	1	Lipperman et al. [44]
	1.4	1	0.07	1	Chen et al. [21]
Diamond	0.44	1	0.45	1	Romijn & Fleck [36]
	0.25	1	0.50	1	Lipperman et al. [44]
	1.28	2	-	-	Huang & Chiang [28]
	1.66	2	-	-	Huang & Gibson [17]

Table 1. Fracture toughness of various lattice topologies. Values of pre-exponent D and exponent d, for scaling law (5).

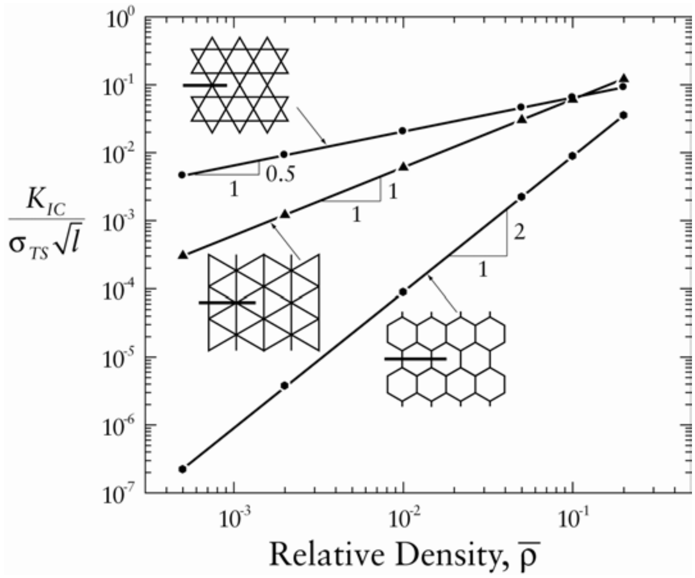

Fig. 5. The predicted mode I fracture toughness K_{IC} plotted as a function of relative density $\overline{\rho} \propto t/l$, for the three isotropic lattices: hexagonal, triangular and Kagome.

It has already been noted that Huang & Lin [13] proposed a mixed-mode fracture criterion for lattice materials, which is a linear combination of K_I / K_{IC} and K_{II} / K_{IIC}. Fleck & Qiu [14] follow a more systematic approach by making use of the boundary layer analysis to determine the mixed-mode fracture envelope for the three isotropic lattices. They find that the fracture locus in $\left(K_I, K_{II}\right)$ space comprises the inner convex envelope of a series of straight lines, corresponding to a particular failure site (Fig. 6). They conclude that the triangular lattice is the most resistant to mode II loading, and the failure envelope is approximately circular in shape. In contrast, the failure envelope for the hexagonal lattice is the most eccentric in shape, with a relatively low value of mode II toughness.

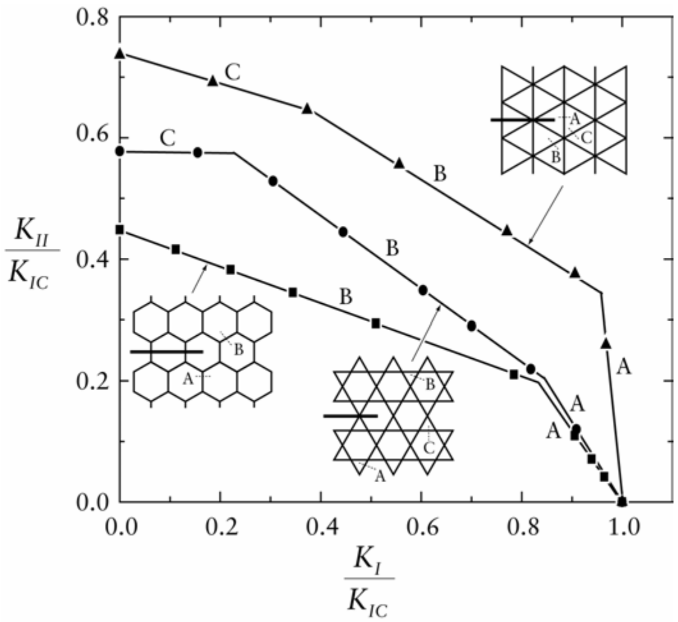

Fig. 6. The failure envelope for mixed-mode loading. The segments of failure surface correspond to the critical sites of failure shown in the insets.

Fleck & Qiu [14] also considered the tensile and shear strengths of centre-cracked plates (CCP) made from each of the three isotropic microstructures. The hexagonal and triangular lattices are flaw-sensitive, with the strength adequately predicted by LEFM for cracks spanning more than a few cells. In contrast, the Kagome microstructure is damage tolerant: for cracks shorter than a transition length its strength is independent of crack length but somewhat below the unnotched strength σ_u of the lattice. At crack lengths exceeding the transition value, the strength decreases with increasing crack length, in accordance with the LEFM estimate. They showed that this transition crack length is given by $K_{IC}^2 / \pi\sigma_u^2$, in agreement with previous studies on fully dense materials [32]. Finally, the authors point out the need to include *T*-stress effects[4] in order to explain the predicted strengths of the centre-cracked plates at sufficiently low stockiness, particularly in the hexagonal lattice.

4.2.1 Extrapolation of 2D results to 3D Lattices

The boundary layer analysis of the fracture toughness problem was also used in the numerical study of Choi & Sankar [35], who investigated mode I and mode II cracks in a three-dimensional (3D) cubic lattice of side length l and composed of square struts of cross-section $t \times t$. It is of concern that Choi & Sankar [35] found values of the coefficient D in Eq. (5) which varied by an order of magnitude depending upon whether they calibrated the equation by varying the cell-wall thickness t or the cell size l.

Recently, Romijn & Fleck [36] have reconsidered the problem of the cubic lattice made from elastic-brittle struts. These authors regard the 3D lattice as a separated stack of 2D square grids each of thickness t. One grid is fastened to the next layer at its joints by out-of-plane bars of length l. This allows them to use toughness calculations for a 2D lattice in order to make predictions for the fracture toughness of the 3D cubic lattice. The fracture toughness of the 3D lattice $K_C^{(3D)}$ is then related to the fracture toughness of the 2D lattice $K_C^{(2D)}$ by $K_C^{(3D)} = (t/l) K_C^{(2D)}$, leading to

$$\begin{aligned} K_{IC}^{(3D)} &= 0.56 (t/l)^2 \sigma_{TS} \sqrt{l} \\ K_{IIC}^{(3D)} &= 0.35 (t/l)^{5/2} \sigma_{TS} \sqrt{l} \end{aligned} \qquad (6)$$

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[4] The second term in the series expansion of the mode I crack tip field is the so-called *T*-stress parallel to the crack plane. The *T*-stress scales linearly with the remote applied stress, but its magnitude depends upon the specimen configuration. The *T*-stress vanishes for mode II loading, by a symmetry argument. For conventional, fully dense, elastic-brittle solids the *T*-stress plays a relatively minor role in influencing the fracture process at the crack tip. In ductile fracture and fatigue, however, the *T*-stress becomes important at short crack lengths and reveals itself in a dependence of inferred fracture toughness upon specimen geometry, see for example [33,34].

4.2.2 Sensitivity of Fracture Toughness to Imperfections

Fracture of lattice materials involves multiple length scales: the crack length at the macroscopic level, the dimensions of the lattice (cell size and strut thickness) at the microstructural level, and even lower length scales such as imperfections at the cell wall level. All the above studies on fracture toughness are for perfect lattices. A detailed treatment of the role of various microstructural imperfections (missing struts, misplaced joints, and wavy cell walls) upon the in-plane effective properties is given in the recent publication by Symons & Fleck [37] for three isotropic lattices: hexagonal, triangular and Kagome. The knockdown in fracture toughness due to misplaced joints is numerically explored in a parallel study by Romijn & Fleck [36] for the three isotropic lattices, as well as two orthotropic topologies: square and diamond-celled.

Both studies conclude that the imperfection sensitivity of modulus and fracture toughness for the lattices examined can be catalogued in terms of the nodal connectivity of each lattice, consistent with the arguments of Deshpande et al. [38]. A connectivity value of 3 struts per joint for the hexagonal lattice is sufficiently low for the struts to deform by bending, and the random misplacement of joints has little effect upon the stress state in the lattice. The high connectivity value of 6 for the triangular lattice causes it to deform by cell-wall stretching. Again, it is insensitive to imperfections: misplaced joints have a negligible effect upon the mechanical properties of the lattice. But the square-celled lattices, like the Kagome, have a transition value of connectivity equal to 4 struts per joint. The response of these lattices can be bending or stretching dominated, depending on the level of imperfection. For instance, under uniform loading the square lattice deforms by strut stretching, but upon introduction of a defect such as a macroscopic crack the struts deform by a combination of strut stretching and strut bending. Thus, the moduli and fracture toughness of these topologies is highly sensitive to imperfections.

5. Atomic Lattice Models for Crack Dynamics

Analytical solutions for crack dynamics in lattices obtained over the last two decades are based on the model where point masses are connected by massless bonds [39]. The first works along these lines were published by Slepyan [40] and by Kulakhmetova et al. [41], who considered the dynamic problem of propagation of a rectilinear crack in periodic lattices with rectangular and triangular cells, respectively (Fig. 7).

These models of the lattice structure can be considered as related to a regular atomic lattice or as a discrete model of a continuous material with periodic structure. With the latter application in mind, a lattice with material bonds of nonzero density has been investigated. This limiting structure can be interpreted as a simplest model of porous material or a material becoming porous in a vicinity of the growing crack tip. It can also be considered as a model of a fibre-reinforced composite or a fabric. Detailed examination of the relation between joint mass and bond mass is undertaken in [42] for the square-celled lattice. This approach assumes that the bonds behave like trusses rather than beams, i.e. they transmit axial forces but do not resist bending moments.

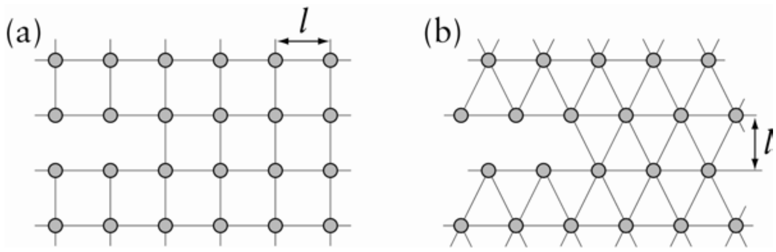

Fig. 7. Atomic lattice models with massless bonds for (a) square and (b) triangular lattices.

A comprehensive review of these lattice models for crack dynamics can be found in the recent monograph by Slepyan [39], where the author discusses models, phenomena, methods, and his overall view on the subject. The approach has inspired the use of techniques such as the representative cell method described in the following section. In addition, it has resulted in a wealth of research on crack kinetics in atomic lattices.

6. Representative Cell Method

A difficulty in the investigation of fracture properties of lattice materials is the need to introduce a large number of degrees of freedom in order to model the material microstructure with a flaw. To overcome this, Lipperman et al. [43,44] have recently developed an analytical method that enables to determine the stress state of an unbounded infinite lattice with an embedded finite-length crack. The analysis technique hinges on the combined use of the structural variation method and the representative cell method [45]. While the latter allows for the analysis of periodic structures under arbitrary loads, by means of the discrete Fourier transform, the former analyses modified structures (the cracked lattice) on the basis of the analysis of the uncracked periodic structure. The material is viewed as an assemblage of identical parallelogram cells defined by the two vectors of the lattice translational symmetry. It was noted by Renton [46] that such a representation is always possible.

In a first study, Lipperman et al. [43] numerically emulate the way in which cracks may nucleate and propagate from a single imperfection until a long and stable crack path is achieved. For the triangular and Kagome lattices, the sequence of broken beams produces straight linear cracks propagating perpendicular to the loading direction. For the hexagonal lattice, however, the crack kinks to a straight line inclined at 60° with respect to the loading direction. A similar deviation was reported by Schmidt & Fleck [30]. For the square lattice two cracks running parallel to the loading direction emerge. Indeed, the stress state at the crack tips of hexagonal and square lattices is characterised by significant mode II deformations. This phenomenon is explained by the relatively small resistance of the lattices to shear deformation.

In a second paper, Lipperman et al. [44] proceed to investigate the fracture toughness of several lattice topologies: triangular, hexagonal, Kagome, square and diamond-celled. Their results for the triangular lattice were found to be in agreement with the exact analytical solution for a semi-infinite crack derived by Slepyan & Ayzenberg-Stepanenko [47] within the context of the atomic lattice models mentioned above. The values of fracture toughness for the hexagonal and Kagome lattices are also in agreement with those obtained by Fleck & Qiu [14] by the boundary layer analysis. For the diamond-celled lattice, they find that the dependence of fracture toughness on stockiness is close to linear, in agreement with Quintana-Alonso and Fleck [15].

The representative cell method is a convenient tool for investigating periodic lattice materials with flaws. It can be applied not only to isolated straight linear cracks but also to more general crack interaction or flaws that are not straight.

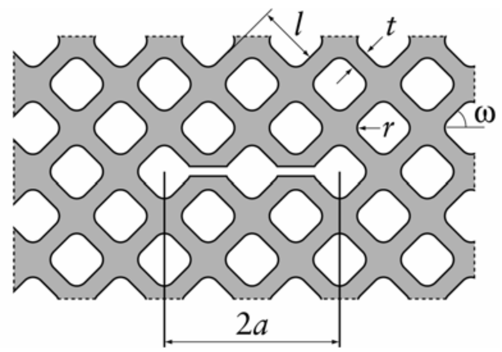

Fig. 8. Macroscopic crack in a porous material with square cells.

Lipperman et al. [48] have recently extended their analysis technique based on the discrete Fourier transform, to investigate the fracture properties of 2D periodic porous material (Fig. 8). They considered materials with periodic voids where the porosity is neither high enough to regard them as lattice materials modelled by systems of beams and plates, nor low enough to model them as an elastic plane with dilute voids. The traditional FE approach for determining the fracture toughness of these materials requires a huge computational effort. The technique presented by Lipperman et al. [48] overcomes this difficulty. They perform a parametric study in order to find the optimal voids shape for maximum fracture toughness at a given porosity. For the specific case of large voids (i.e. high porosity) when the model of a material consisting of beam elements becomes valid the results are found to be in agreement with the FE data of Fleck & Qiu [14] for 2D hexagonal lattices.

The method employed by Lipperman et al. [48] can be used to evaluate the fracture toughness of more complicated microstructures. For example, their parametric study of void shape could serve to investigate the effect upon fracture toughness of varying the core angle ω of the diamond-celled lattice (Fig. 8).

The analytical and numerical work reviewed thus far provides a useful foundation for subsequent investigations of the fracture properties of lattice materials. The next section highlights the few experimental studies identified in the literature.

7. Experimental Studies on Fracture Toughness

To date, experimental studies on brittle lattice materials have focused on determining the strength of intact lattices made from glasses or ceramics. Scheffler & Colombo [49] provide a comprehensive summary. But experimental research into the fracture toughness of brittle lattice materials is scarce. Huang & Gibson [17] have determined the fracture toughness of a cordierite ($2MgO$-$2Al_2O_3$-$5SiO_2$) lattice with square cells by testing single-edge notched beams in three-point bending, as shown in Fig. 3. The fracture toughness of the diamond-celled lattice was calculated from the measured failure load P_{cr}, using the *K*-calibration given in [50] for an isotropic continuum. Recently, Quintana-Alonso et al. [29] have performed tests over a much wider range of t/l and found that K_{IC} scales linearly with relative density.

8. Concluding Remarks

Since the pioneering work of Ashby [10], who provided the first analytical estimate of the fracture toughness of a planar lattice, only a limited amount of research has been conducted on the fracture properties of lattice materials. Attempts to model the fracture behaviour of lattice materials, making use of generalised continuum theory, finite element analysis, and novel techniques such as the representative cell method, have met with various degrees of success.

The study of orthotropic lattices, however, has led to contradictory results in the literature. For example, the correct functional dependence of fracture toughness upon stockiness has been debated. The experimental data available is too limited and inconclusive.

All the studies reviewed herein have focused on determining the effective fracture properties of brittle lattices. In practice, lattice materials are commonly loaded in a sandwich panel configuration with stiff and strong face-sheets. The flaw sensitivity of these *lattice structures* has not yet been explored.

In view of the above, the following two directions of research are expected to be fruitful: (i) the fracture toughness of orthotropic lattices, and (ii) the crack-sensitivity of a sandwich panel containing a defective lattice core.

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