The impact of sand slugs against beams and plates: coupled discrete/continuum calculations

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Abstract

Coupled finite element (FE) and discrete particle simulations of sand columns (slugs) comprising identical particles impacting end-clamped monolithic and sandwich beams as well as plates are used to elucidate the sand-structure interaction. The majority of the calculations are performed using slugs comprising identical circular cylindrical particles impacting beams. These calculations demonstrate that the loading due to the sand was primarily inertial with little fluid-structure interaction effects, i.e. the momentum transmitted to the structures was approximately equal to the momentum of the incoming sand slug. The loading and response of the structures is well characterised by accounting for just two parameters: the initial momentum of the sand column and the loading time given by the ratio of the height of the column and the initial velocity of the sand particles. Sandwich beams with thick and strong cores were shown to significantly outperform monolithic beams of equal areal mass. This performance enhancement is shown to result from the "sandwich effect" whereby the high bending strength of sandwich beams gives superior performance when the beam deflections are less than the core thickness. The main sand-structure interaction mechanisms elucidated by the beam calculations are shown to hold in a three-dimensional setting where slugs comprising spherical sand particles are impacted against edge-clamped circular monolithic and sandwich plates. The fluid-structure interaction effects were again found to be negligible and sandwich plates with thick strong cores out-performed monolithic plates of equal areal mass. Finally, we present a foam analogy where impact of the sand particles is shown to be equivalent to the impact of an equivalent crushable foam projectile. The calculations on the equivalent projectile are significantly less intensive computationally and yet give predictions to within 5% of the full discrete particle calculations for the monolithic and sandwich beams and plates. These foam projectile calculations suggest that "model" materials can be used to simulate the loading by sand particles within a laboratory setting.

Keywords: fluid-structure interaction, discrete/continuum coupling, dynamic loading.

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1.   Introduction

The air and water blast resistance of structures has recently received considerable attention with the aim of designing lightweight blast resistant structures. Several recent theoretical studies have shown that sandwich structures subjected to water blast loading outperform monolithic structures of equal mass, see for example Fleck and Deshpande (2004) and Xue and Hutchinson (2004). These predictions have been confirmed experimentally by Wadley et al (2008) and Wei et al (2009). This enhanced performance is mainly due to fluid-structure interaction effects with a smaller fraction of the impulse being transmitted into sandwich structures compared to their monolithic counterparts. On the other hand, under air blast loading, sandwich structures provide only marginal benefits over monolithic structures as the fluid-structure interaction effects are significantly less important (Kambouchev et al., 2007). The main aim of this study is to investigate the relative performance of monolithic and sandwich structures subjected to loading by the impact of high velocity granular media that are accelerated by the explosion of shallow buried explosives (landmines).

The prototypical problem is sketched in Fig. 1 where the sandwich structure represents the under-body of a vehicle. The phenomena in play are complex and can typically be separated into three sequential phases: (i) The transfer of impulse from the explosive to surrounding soil, leading to the formation of a dispersion of high velocity particles, (ii) the propagation (with spreading) of the expanding soil ejecta and (iii) impact of the soil ejecta with the structure leading to momentum transfer to the protection system (i.e. the soil-structure interaction). The focus of this study is on the third phase; the interaction of high velocity granular media with a structure and the subsequent response of the structure.

Direct experimental characterizations of buried explosive events (Bergeron et al., 1998; Braid, 2001; Weckert and Andersen, 2006; Neuberger et al., 2007) have been the main focus of research to-date and have led to the development of empirical models to quantify impulsive loads imposed by the soil ejecta (Westine et al., 1985) as well as structural design codes such as those proposed by Morris (1993). The predictive capability of these empirical models is limited and cannot be extrapolated to new structural concepts such as the sandwich structure sketched in Fig. 1. Most research at developing a predictive capability has focussed on developing appropriate constitutive models for the soil that can be implemented within Eulerian numerical codes. These Eulerian codes can then be coupled to Lagrangian finite element (FE) calculations to simulate the structural response. Readers are referred to Grujicic et al. (2006) and Grujicic et al. (2008) for a detailed analysis of soil models used to simulate landmine explosions. Notable among these are the so-called three phase model of Wang et al. (2003) and Wang et al. (2004) which is a modified Drucker and Prager (1952) model and the *porous-material/compaction* model developed by Laine and Sandvik (2001). The soil models listed above are restricted to a regime where the packing density of the soil is sufficiently high that the particle-particle contacts are semi-permanent. While these models are appropriate during the initial stages of a buried explosion or during an avalanche or land slide, their applicability when widely dispersed particles impact a structure is questionable. Deshpande et al. (2009) modified a constitutive model of Bagnold (1954) to develop a continuum soil model applicable to soils in both their densely packed and dispersed states. However, the successful implementation of this model within a coupled Eulerian-Lagrangian computational framework has been elusive because of well known computational

problems associated with the analysis of low density particle sprays; see Wang et al. (2004) for a discussion of these numerical issues.

An alternative modelling strategy has recently been employed by Borvik et al. (2011) and Pingle et al. (2011) wherein the low density soil is treated as an aggregate of particles with the contact law between the particles dictating the overall aggregate behaviour. This approach has several advantages: (i) there is no need to make a-priori assumptions about the constitutive response of the aggregate (this becomes an outcome of the simulations), (ii) it provides a fundamental tool to study the essential physics of the sand-structure interaction and (iii) given that the sand is represented by a discrete set of particles we do not face the usual numerical problems associated with solving the equations associated with the equivalent continuum descriptions. Pingle et al. (2011) have recently investigated the response of rigid targets impacted by columns of particles. This rather idealised but fundamental fluid-structure interaction (FSI) problem is the "sand-blast" analogue to the classical water-blast FSI problem studied by Taylor (1963). On the other hand, Borvik et al. (2011) analysed the response of monolithic plates loaded by spherically expanding sand shells and compared their predictions with measurements. Their study demonstrated the capabilities of this methodology in modelling complex sand-structure interaction events. However, given the complexity of the boundary value problem analysed, it was hard to extract a more fundamental understanding of the key phenomena at play.

1.1   Approach and scope
Our aim in this study is to consider a relatively simple loading scenario involving a spatially uniform column of discrete, identical particles impacting monolithic and sandwich structures. This column comprising either identical circular cylindrical or spherical particles will subsequently be referred to as a sand slug. While this slug is not directly representative of the ejecta created during a landmine explosion, it enables us to develop a physical understanding of the phenomena at play and compare the relative performance of monolithic and sandwich structures for a well characterised loading scenario.

The outline of the paper is as follows. First, the methodology employed to couple the discrete and continuum Lagrangian FE calculations is described. Next, the response of clamped monolithic and sandwich beams impacted by sand slugs comprising identical circular cylindrical particles is discussed. Finally we extend the analysis to three-dimensions (3D) and present the response of clamped circular plates loaded by cylindrical sand slugs comprising identical spherical particles.

2.   Summary of the plane strain coupled discrete element/finite element methodology
The plane strain boundary value problems are sketched in Fig. 2 and comprise a slug of a low density granular medium (subsequently referred to as sand) which impacts a clamped beam. The granular medium was modelled as a set of two-dimensional (2D) discrete identical circular cylindrical particles using the GRANULAR package in the multi-purpose molecular dynamics code LAMMPS (Plimpton, 1995) while the beams were modelled using a 2D finite strain Lagrangian finite element (FE) framework. We shall first briefly describe the discrete particle and FE numerical methods and then provide details of the coupling of these two techniques.

2.1   Discrete element calculations

The granular medium was modelled as 2D cylindrical particles of diameter D lying in the $x_1 - x_2$ plane (thickness b in the x_3-direction). The granular package in LAMMPS is based on the soft-particle contact model introduced by Cundall and Strack (1979) and extended to large scale simulations by Campbell and co-workers (Campbell and Brennen, 1985; Campbell, 2002). A schematic of the soft-particle contact model is shown in Fig. 3 and comprises a linear spring K_n and linear dashpot with damping constant γ_n, governing the normal motion and a linear spring K_s and Coulomb friction coefficient μ, governing the tangential motion during the contact and deformation of two particles of mass m_p. Define unit vectors $\hat{e}_n$ and $\hat{e}_s$ such that $\hat{e}_n$ is in the direction of the outward normal to the particles along the line connecting the centres of the two particles and $\hat{e}_n \times \hat{e}_s$ is a unit vector in the x_3 direction. The force acting on each particle is then given as

$$F = \begin{cases} F_n \hat{e}_n - F_s \hat{e}_s & \text{if} \quad \delta_n \leq 0 \\ 0 & \text{otherwise} \end{cases} \tag{2.1}$$

where r is the distance between the particle centres $\delta_n = r - D$ is the interpenetration. The normal force is given as

$$F_n = K_n \delta_n + m_{eff} \gamma_n \dot{\delta}_n \tag{2.2}$$

where m_{eff} is the effective or reduced mass of the two contacting bodies. In the calculations presented here impacts were either between particles or between particles and the beams with $m_{eff} = m_p / 2$ and m_p in these two cases, respectively. It now remains to specify the tangential force F_s. Define $\dot{\delta}_s$ as the tangential displacement rate between the contacting particles. We then stipulate F_s via an "elastic-plastic" relation so as to simulate Coulomb friction, i.e.

$$\dot{F}_s = \begin{cases} K_s \dot{\delta}_s & \text{if} \quad |F_s| < \mu |F_n| \text{ or } F_s \dot{\delta}_s < 0 \\ 0 & \text{otherwise} \end{cases} \tag{2.3}$$

As discussed by Bathurst and Rothenburg (1988) the Poisson's ratio of the solid is related to the ratio K_s / K_n, while K_n is a function of the Young's modulus. Furthermore, it is clear that the damping constant γ_n, will determine the loss of energy during normal collisions and will therefore be directly related to the coefficient of restitution e for normal collisions. Consequently, appropriate values of γ_n can be determined from the known or measured coefficient of restitution via the relation

$$e = \exp\left[-\frac{\pi}{\left(8K_n / (\gamma_n^2 m_p) - 1\right)^{1/2}} \right]. \tag{2.4}$$

Note that this particle interaction model leads to a collision time for individual binary collisions t_e given by

$$t_e == -2 \frac{\ln(e)}{\gamma_n}. \tag{2.5}$$

Thus, in the limit of plastic collisions with $e \to 0$, the contact time $t_e \to \infty$. This simplest contact model has the feature that the contact properties can be readily related to the coefficient of restitution e used in most analytical treatments.

The calculations were performed using the GRANULAR package in the molecular dynamics code LAMMPS. The Newton equations for both the translational and rotational motions of the particles were integrated using a Verlet time-integration scheme (i.e. Newmark-Beta with $\beta = 0.5$). The time-step was typically taken to be about $t_e / 10$ in order to ensure accurate integration of the contact relations, Eqs. (2.1)-(2.3) and equal to that used in the finite element calculations described below.

2.2   Two-dimensional finite element calculations

The 2D plane strain calculations were performed using an updated Lagrangian finite element (FE) scheme with the current configuration at time *t* serving as the reference. The coordinate x_i denotes the position of a material point in the current configuration with respect to a fixed Cartesian frame, and v_i is the velocity of that material point. For the plane strain problem under consideration, the principle of virtual power (neglecting effects of gravity) for a volume V and surface S is written in the form

$$\int_V \sigma_{ij} \delta \dot{\varepsilon}_{ij} dV = \int_{S_T} T_i \delta v_i dS - \int_V \rho \delta \dot{v}_i \delta v_i dV \qquad (2.6)$$

where σ_{ij} is the Cauchy stress, $\dot{\varepsilon}_{ij} \equiv 0.5 \left(v_{i,j} + v_{j,i} \right)$ is the strain rate, T_i the tractions on the surface $S_T \in S$ due to the impacts of the particles while ρ is the material density in the current configuration. The symbol δ denotes arbitrary virtual variations in the respectively quantities. A finite element discretisation based on linear, plane strain three node triangular elements (i.e. constant strain triangles) is employed. When the finite element discretisation of the displacement field is substituted into the principle of virtual power (2.6) and the integrations are carried out, the discretised equations of motion are obtained as

$$\mathbf{M} \frac{\partial^2 \mathbf{U}}{\partial t^2} = \mathbf{F} \qquad (2.7)$$

where $\mathbf{U}$ is the vector of nodal displacements, $\mathbf{M}$ is the mass matrix and $\mathbf{F}$ is the nodal force vector. An explicit time integration scheme based on the Newmark β-method with $\beta = 0$ was used to integrate Eq. (2.7) to obtain the nodal velocities and the nodal displacements. A lumped mass matrix is used in (2.7) instead of a consistent mass matrix, since this is preferable for explicit time integration procedures, for both accuracy and computational efficiency.

2.3   Coupling of the discrete element and finite element calculations

The coupling between the discrete and finite element calculations was carried out as follows. At time t, contact between the particles and the beam in its current configuration was detected. The displacement δ_n is defined as the $\delta_n = r - D/2$, where r is the distance between the particle centre and the contact point on the beam while $\dot{\delta}_s$ is the tangential displacement rate of the particle relative to the beam measured. Note that the rate $\dot{\delta}_n$ is the relative velocity of the particle and the point of contact on the beam surface along the normal to the beam surface at the point of contact on the beam[†]. The normal and tangential contact forces are then calculated using Eqs. (2.1) - (2.3). These normal and tangential forces are then transformed to

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[†] The velocity of any point on the beam is calculated by interpolating the nodal velocities using the shape functions of the constant strain triangle elements.

the global coordinate system and a vector of equivalent nodal forces determined for each element on the surface of the beam. These vectors of elemental nodal forces are then inserted into the vector of global nodal forces $\mathbf{F}$ in Eq. (2.7). Subsequently, the discrete and finite element equations were integrated as described in Sections 2.1 and 2.2 and the new positions and velocities of the particles in the discrete calculations and material points in the finite element calculations determined at time $t + \Delta t$.

3.   The monolithic and sandwich beam boundary value problems

The two plane strain boundary value problems investigated in this study are sketched in Figs. 2a and 2b. In both problems a slug of height H and width $2a$ impacts the clamped beams centrally and normally at time $t = 0$. The slug comprises discrete identical cylindrical particles of diameter D made from of solid material of density ρ_s and randomly packed with uniform spatial relative density $\bar{\rho}$. Each particle has an initial velocity v_o in the negative x_2 direction.

In Fig. 2a shows a sketch of a monolithic beam of span $2L$ and made from a material with a density ρ_m. The beam has a thickness $2h$ and is fully clamped at its two ends. The equivalent clamped sandwich beam of span $2L$ sketched in Fig. 2b comprises two identical face sheets of thickness h_f also made from the same material and separated by a compressible foam core of density ρ_c and thickness c. The areal mass m_b of the monolithic and sandwich beam are equal such that

$$m_b = 2\rho_m h = 2h_f \rho_m + \rho_c c . \qquad (3.1)$$

In order to reduce the variables needed to specify the sandwich geometry we restrict attention in this study to the situation where mass of the sandwich is divided equally between the two faces and the core, i.e.

$$h_f \rho_m = \rho_c c = \frac{m_b}{3} \qquad (3.2)$$

This mass distribution was shown to be near optimal for sandwich beams under impulsive loading (Xue and Hutchinson, 2004). Thus, given a monolithic beam of areal mass M there remain only two free geometric variables for the sandwich beam (ρ_c, c) that are constrained via Eq. (3.2).

In all the calculations presented here, the sandwich and monolithic beams have a half-span $L = 0.5\,\mathrm{m}$. The monolithic beam is made of DH 36 steel and thickness $2h = 50\,\mathrm{mm}$ corresponding to an areal mass $m_b = 400\,\mathrm{kgm^{-2}}$ for an assumed steel density $\rho_m = 8000\,\mathrm{kgm^{-3}}$. Two sandwich geometries with aspect ratio $c/L = 0.11$ and 0.33 are investigated. For each for these geometries the thickness of the sandwich beam face sheets and core density follows from Eq. (3.2). Only half the beam spans were modelled with symmetry boundary conditions imposed along $x_1 = 0$ and all degrees of freedom constrained along $x_1 = L$ in order to simulate the clamped boundary conditions. The beams were discretised using quadrilaterals comprising four constant strain triangles (Peirce et al., 1984) with elements of size $h/15$ and $h_f/8$ in the monolithic and sandwich beams, respectively.

3.1 Material properties

The monolithic beam and sandwich beam face sheets were made from DH36 steel. This steel was modelled as an elastic-plastic J2 flow theory solid with a Young's modulus $E = 200\,\text{GPa}$, Poisson's ratio $\nu = 0.3$ and density $\rho_m = 8000\,\text{kgm}^{-3}$. The rate dependent uniaxial yield strength of the steel was specified via the relation (Vaziri et al., 2007)

$$\sigma_Y = Y\left[1 + 1.5(\varepsilon_{eff}^{\text{P}})^{0.4}\right]\left[1 + 0.015\ln\left(\frac{\dot{\varepsilon}_{eff}^{\text{P}}}{\dot{\varepsilon}_0}\right)\right], (3.3)$$

where $Y = 470\,\text{MPa}$, $\dot{\varepsilon}_0 = 1\,\text{s}^{-1}$ and $\dot{\varepsilon}_{eff}^{\text{P}}$ is the von-Mises equivalent plastic strain rate.

The core in the sandwich beams was modelled a homogeneous compressible visco-plastic orthotropic foam-like material following Tilbrook et al. (2006). Assume the orthotropic axes x_i of the core are aligned with the axes of the beam as sketched in Fig. 2, i.e x_1 and x_2 are aligned with the longitudinal and transverse directions, respectively. Introduce the stress and plastic strain matrices in the usual way as

$$\boldsymbol{\sigma} = (\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6)^T \equiv (\sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{13}, \sigma_{23}, \sigma_{12})^T, \quad (3.4)$$

and

$$\boldsymbol{\varepsilon}^P = (\varepsilon_1^p, \varepsilon_2^p, \varepsilon_3^p, \varepsilon_4^p, \varepsilon_5^p, \varepsilon_6^p)^T \equiv (\varepsilon_{11}^p, \varepsilon_{22}^p, \varepsilon_{33}^p, \varepsilon_{13}^p, \varepsilon_{23}^p, \varepsilon_{12}^p)^T, \quad (3.5)$$

respectively. Complete decoupling of material response between the orthogonal material directions is assumed and we define the plastic strain rate $\dot{\varepsilon}_i^p$ via an overstress relation as

$$\dot{\varepsilon}_i^p = \begin{cases} \left(\dfrac{|\sigma_i| - Y_i(\varepsilon_i^p)}{\eta}\right) & \text{if } |\sigma_i| > Y_i(\varepsilon_i^p) \\ 0 & \text{otherwise,} \end{cases} \quad (3.6)$$

where the yield strength $Y_i(\varepsilon_i^p)$ is a function only of the plastic strain ε_i^p and η is the material viscosity. The total strain rate $\dot{\varepsilon}_i$ is obtained by supplementing the above anisotropic plasticity model with isotropic elasticity such that

$$\dot{\varepsilon}_i = L_{ij}\dot{\sigma}_j + \dot{\varepsilon}_i^p \text{sign}(\sigma_i) \qquad \text{(summation over } j). \quad (3.7)$$

In the case of isotropic elasticity, the compliance matrix L_{ij} of the core material is specified in terms of the Young's modulus E_c and Poisson's ratio ν_c as

$$\mathbf{L} = \begin{pmatrix} 1/E_c & -\nu_c/E_c & -\nu_c/E_c & 0 & 0 & 0 \\ & 1/E_c & -\nu_c/E_c & 0 & 0 & 0 \\ & & 1/E_c & 0 & 0 & 0 \\ & & & 2(1+\nu_c)/E_c & 0 & 0 \\ & \text{sym} & & & 2(1+\nu_c)/E_c & 0 \\ & & & & & 2(1+\nu_c)/E_c \end{pmatrix} (3.8)$$

We employ an isotropic elastic response for simplicity; this suffices as the core response is dictated by the plastic branch. The above elastic-plastic constitutive relation is expected to be adequate to model sandwich cores such as the square-honeycomb core or the corrugated core; see for example Xue et al. (2005).

In all the calculations reported here we assume an isotropic plastic response in the sense that the all the strengths Y_i are assumed to be equal and strain hardening is neglected in all directions other than the transverse direction. The transverse strength Y_2 is assumed to be independent of the plastic strain ε_2^p up to a nominal densification strain ε_D: beyond densification a linear hardening behaviour is assumed with a very large tangent modulus $E_t = 0.1E$. The core has a Young's modulus $E_c = \bar{\rho}_c E$ where the relative density of the foam core $\bar{\rho}_c \equiv \rho_c / \rho_m$ (assuming that the foam is made of the same solid material as the face sheets) and a Poisson's ratio $v_c = 0.25$; numerical experiments confirmed that the predictions are insensitive to the elastic stiffness of the core. The nominal densification strain ε_D is related to the core relative density via the relation (Ashby et al., 2000)

$$\varepsilon_D = 1 - 1.5\bar{\rho}, \qquad (3.9)$$

while the transverse strength $Y_2 \equiv \sigma_c$ is varied in the parametric studies reported subsequently. Finally, we note that the viscosity η of the core was chosen such that the shock width (Radford et al., 2005)

$$l = \frac{\eta \varepsilon_D}{\rho_c v_o} = \frac{c}{10}, \qquad (3.10)$$

with v_o interpreted as the initial impact velocity of the sand particles. This prescription ensures that the shock width is always much less than the core depth yet is larger than the mesh size. Note that large gradients in stress and strain occur over the shock width and thus a mesh size smaller than l is required to resolve these gradients accurately.

3.2 Properties of the impacting particles
The particles properties were chosen so as to mimic silica sand particles. Unless, otherwise specified the following parameters were employed in all the calculations presented here. The identical cylindrical particles of diameter $D = 200\,\mu m$ and thickness $b = 1\,mm$ (in the x_3 direction) had a solid material density $\rho_s = 2700\,kg\,m^{-3}$. The normal stiffness between the particles was taken to be $K_n = 7300\,kNm^{-1}$ and the co-efficient of restitution for both impacts between the particles and the particles and the beam assumed to be $e = 0.7$ (i.e. the value of γ_n for inter-particle contacts was twice that for impacts between the particles and the beam). Following Silbert et al. (2001) we fixed the ratio $K_s / K_n = 2/7$ and the reference value of the friction co-efficient was assumed to be $\mu = 0.7$.

4. Impact against the monolithic beam
Consider the monolithic end-clamped beam impacted by a sand slug at time $t = 0$ as sketched in Fig. 2a. Define the non-dimensional loading parameters

$$\bar{a} \equiv \frac{a}{L}, \quad \bar{m} = \frac{\bar{\rho}\rho_s H}{m_b}, \quad \bar{\rho}, \quad \text{and} \quad \bar{I}_o \equiv \frac{I_o}{m_b\sqrt{\sigma_Y / \rho_m}} \qquad (4.1)$$

where $I_o = \bar{\rho}\rho_s H v_o$ is the areal momentum of the impacting slug, $m_b = 2\rho_m h$ is the areal mass of the beam. Dimensional analysis dictates that for a given set of beam

material and sand properties, the deflection w of the mid-span of the beam has the functional form

$$\bar{w} \equiv \frac{w}{L} = f\left(\bar{t}, \frac{h}{L}, \frac{\rho_m}{\rho_s}, \bar{a}, \bar{I}_o, \bar{m}, \bar{\rho}\right). \qquad (4.2)$$

where $\bar{t} \equiv t / \left(L\sqrt{\rho_m / \sigma_Y}\right)$ is the time normalised by the structural response time of a plastic string of length $2L$ and made from a material of density ρ_m and yield strength σ_Y. Recall that all calculations are presented for a beam of aspect ratio $h / L = 0.05$ and a parametric study is reported for the sensitivity of the response to the loading parameters $\bar{a}$, $\bar{I}_o$, $\bar{m}$ and $\bar{\rho}$. Unless otherwise specified these parameters had the following reference values: $\bar{a} = 0.2$, $\bar{\rho} = 0.2$, $\bar{m} = 0.405$ (sand slug height $H = 300\,\mathrm{mm}$). We note in passing that consistent with the findings of Pingle et al. (2011) for sand impacting a rigid target, the contact properties of sand particles have a very minor effect on the response of the beam; varying K_n by two orders of magnitude and changing e and μ from by factors of five had less than a 2% effect on the predicted beam deflections.

4.1   Effect of sand momentum

Predictions of the normalised mid-span deflection $\bar{w}$ versus normalised time $\bar{t}$ response of the beam are plotted in Fig. 4a for three selected values of the normalised areal momentum of the sand $\bar{I}_o$ (the momentum was varied by keeping all parameters fixed at their reference values and increasing the sand impact velocity from $v_o = 100\,\mathrm{ms^{-1}}$ to $400\,\mathrm{ms^{-1}}$). The maximum deflections increase with increasing $\bar{I}_o$ and all occur at $\bar{t} \approx 1$. The maximum normalised deflections $\bar{w}_{\max} \equiv w_{\max} / L$ are summarised in Fig. 4b as a function of $\bar{I}_o$ while snapshots showing the deformation of the sand slug and the beam at selected times are included in Fig. 5 for the $\bar{I}_o = 0.67$ case. In the initial stages of the deformation, the sand slug compacts against the beam with a near planar densification front propagating through the slug from the impacted to the distal end of the slug. With increasing time, there is significant lateral spreading of the slug and the densification front no longer remains planar. This densification front reaches the distal end of the slug at $t v_o / H \approx 1$ and subsequently the beam deflection continues to increase due to its acquired momentum while the sand flows laterally. Elastic vibrations of the beam continue after the beam has attained its peak deflection.

We proceed to quantify the loading of the beam due to the impact of the sand slug. For this purpose we calculate the pressure p exerted by the sand particles on the beam as follows. At any time t, there are M sand particles in contact with one of the finite elements on the impacted surface of the beam. In the undeformed state, the mid-point of that element on the surface of the beam had a x_1 coordinate X_m. The pressure at location X_m at time t is then defined as

$$p(X_m, t) = \frac{\sum\limits_{i=1}^{M} F_2^i}{l_e} \qquad (4.3)$$

where F_2^i is the contact force in the x_2 direction between the ith sand particle and the beam and l_e is the undeformed length of the finite element on the surface of the beam. The normalised pressure $p/(\bar{\rho}\rho_s v_o^2)$ is plotted in Fig. 6 for the $\bar{I}_o = 0.67$ case as a function of the spatial coordinate X, where X denotes the x_1 cordinate a material point on the beam surface in its underformed configuration, i.e. $X = 0$ is at the beam mid-span and $X = L$ is at the supports. Results are shown in Fig. 6 for three selected times such that at $\bar{t} = 0.1$ and 0.25, the sand slug has not fully compacted while at $\bar{t} = 1$, the beam has reached its peak deflection with the sand flowing laterally as seen in Fig. 5. It is clear from Fig. 6 that while the sand slug is compacting, the spatial pressure profile over the beam surface is approximately rectangular with the pressure mainly acting over the width of undeformed slug, i.e. over a width $2a$ at the mid-span. The magnitude of this pressure is initially approximately $\bar{\rho}\rho_s v_o^2$ but decreases with increasing time. After the sand slug has fully compacted, it mainly flows laterally, and then exerts a negligible pressure on the beam. Given that the pressure on the beam is approximately spatially uniform over a central patch of width $2a$ and zero outside this patch we define an average pressure

$$\bar{p}(t) = \frac{1}{2a} \int_0^{2L} p\,dX \,. \tag{4.4}$$

The temporal variation of the normalised pressure $\bar{p}/(\bar{\rho}\rho_s v_o^2)$ is shown in Fig. 7a for two values of $\bar{I}_o$. Here we chose to plot the pressure as function of the normalised time $\hat{t} \equiv t v_o / H$ for reasons that will be clarified subsequently. We make two main observations:

(i)   The pressure $\bar{p}/(\bar{\rho}\rho_s v_o^2) \approx 1$ at $t = 0$ in all cases and then decreases slowly with increasing time.

(ii)   The pressure drops to nearly zero at $\hat{t} \approx 1$ in all cases.

We rationalise these observations as follows. First, as discussed in Pingle et al. (2011), the loading due to the sand is mainly inertial and hence scales as $\bar{\rho}\rho_s \Delta v^2$, where Δv is the relative velocity between the sand and the beam. At time $t = 0$, the beam is stationary and hence $\Delta v = v_o$ resulting in a pressure $\bar{p}/(\bar{\rho}\rho_s v_o^2) \approx 1$. With increasing time, the velocity of the beam increases which implies that Δv decreases. Hence the pressure $\bar{p}$ gradually reduces. Second, the distal end of the sand slug reaches the beam at time $t \approx H/v_o$ and the subsequent motion of the sand is mainly in the lateral direction. Thus, the pressure drops dramatically at $\hat{t} \approx 1$. The corresponding predictions of the normalised momentum I_t/I_o transmitted into the beam are plotted in Fig. 7b, where

$$I_t(t) = \int_0^t \bar{p}\,dt \,. \tag{4.5}$$

The temporal variation of the momentum I_t displays two "knees". One knee occurs at $\hat{t} \approx 1$ where after the rate $\dot{I}_t$ decreases significantly. The transmitted momentum I_t continues to rise until $I_t/I_o \approx 1$ at which point the second knee occurs and I_t remains constant thereafter. It is clear that the first knee corresponds to the instant when the distal end of the sand slug reaches the beam and there is a significant drop in the

pressure exerted by the sand on the beam. Subsequently, the lateral flow of the sand along the deformed beam exerts a small pressure as the sand acquires a small velocity in the positive x_2 due to the deformed profile of the beam (see snapshots in Fig. 5). This small pressure results in the gradual increase in I_t. After the beam reaches its maximum deflection and rebounds back elastically, the sand loses contact with the beam (see Fig. 5) and hence I_t plateaus out. Recall that the beam attains its peak deflection at $\bar{t} \approx 1$, irrespective of the value of I_o. Thus, the second knee in Fig. 7b occurs values of $\hat{t}$ that increase with increasing $\bar{I}_o$ (in these simulations v_o increased with increasing $\bar{I}_o$).

4.2   Parametric study
We investigate here the sensitivity of the maximum deflection $\bar{w}_{\max}$ to the parameters $\bar{m}$, $\bar{\rho}$ and $\bar{a}$.

First consider the effect of $\bar{m}$ and $\bar{\rho}$. The effect of these two parameters is best understood by examining the ratio $\bar{\tau}$ of the loading time due to the sand slug and the response time of the beam. Recall that the response time of the beam is approximately $L\sqrt{\rho_m / \sigma_Y}$ while the loading time is H / v_o. Thus, $\bar{\tau}$ is given by

$$\bar{\tau} = \frac{\bar{m}^2}{\bar{\rho}\bar{I}_o} \left(\frac{h}{L} \frac{\rho_m}{\rho_s} \right) \qquad (4.6)$$

Qiu et al. (2003) as well as Xue and Hutchinson (2003) have demonstrated that the dynamic response of the beam is only dependent on $\bar{I}_o$ for values of $\bar{\tau} < 0.01$, i.e. the loading is impulsive and the response of the beam is independent of the applied pressure. For higher values of $\bar{\tau}$, maximum deflections decrease with increasing $\bar{\tau}$ as the response now depends on both the applied pressure and the impulse. We shall show subsequently that effect of both $\bar{m}$ and $\bar{\rho}$ is adequately captured via the single parameter $\bar{\tau}$.

Calculations were conducted by varying $\bar{m}$ over the range 0.05 to 1.22 for two selected values of $\bar{I}_o$ with $\bar{\rho}$ and $\bar{a}$ both held fixed at the reference values of 0.2. The predictions of $\bar{w}_{\max}$ are summarised in Fig. 8 (the filled squares) in terms of $\bar{\tau}$: the maximum deflections are insensitive to $\bar{m}$ for $\bar{\tau} < 0.01$ and then decrease with increasing $\bar{m}$ or $\bar{\tau}$. Next calculations were performed with $\bar{m}$ and $\bar{a}$ held fixed at their reference values and the effect of $\bar{\rho}$ ($0.1 \le \bar{\rho} \le 0.5$) investigated again for $\bar{I}_o = 0.67$ and 1.0. The predictions of $\bar{w}_{\max}$ are included in Fig. 8 (open circles). It is evident that the dependence of $\bar{w}_{\max}$ on both $\bar{m}$ and $\bar{\rho}$ is adequately captured via the single parameter $\bar{\tau}$.

The dependence of the maximum deflections $\bar{w}_{\max}$ on the normalised width $\bar{a}$ of the sand slug is summarised in Fig. 9a for two selected values of $\bar{I}_o$ and all other parameters held fixed at their reference values. Similar to the impulsive loading results of Qiu et al. (2005), the deflections increase with increasing $\bar{a}$ for a given value of $\bar{I}_o$. Snapshots showing the deformation of the beam and sand slug are

included in Fig. 9b for the case with $\overline{I}_o = 0.67$ and $\overline{a} = 0.8$. The deformation mode is very similar to that seen in Fig. 5 for the $\overline{a} = 0.2$ case. Thus, the increase in the deflection is not due to a change in the deformation mode but rather only because the total momentum of the sand slug increases with increasing $\overline{a}$ with all other parameters held fixed.

4.3   The effect of the lateral spreading of the sand
In order to investigate the effect of the spreading of the sand, we perform calculations by including a constraint in the simulations such that no sand particles can move into the region $x_1 > a$, i.e. the width of the sand slug remains constant during the entire deformation history. Time snapshots showing the deformation of the beam and sand slug from a simulation employing this constraint are included in Fig. 10a for $\overline{I}_o = 0.67$ and the other parameters held fixed at their reference values. The deformation of the sand slug in Fig. 10a is reminiscent of the impact of a foam on a target with a planar densification front travelling from the impacted end of the slug towards the distal end (Radford et al., 2005). After the densification front reaches the distal end it reflects as a tensile wave and the sand spalls and travels in the negative x_2 direction.

Predictions of the variation of $\overline{w}_{max}$ with $\overline{I}_o$ are included in Fig. 10b for two values $\overline{\rho}$ and all other parameters held fixed at their reference values. The figure includes two types of simulations: in the *unconstrained* simulations lateral spreading of the sand is permitted while in the *constrained* simulations no spreading of the sand into the region $x_1 > a$ is permitted. The predictions of $\overline{w}_{max}$ for the $\overline{\rho} = 0.2$ case are nearly identical in the constrained and unconstrained cases while for $\overline{\rho} = 0.45$, $\overline{w}_{max}$ is about 5% higher in the constrained case compared to the unconstrained case. These results indicate (i) the lateral spreading does not have a significant effect on the deformation of the beam and (ii) the additional deflection in the constrained $\overline{\rho} = 0.45$ case is due to the stronger bounce-back of the sand at higher values of $\overline{\rho}$ as discussed by Pingle et al. (2011). This stronger bounce-back results in a higher transmitted momentum into the beam and a slightly enhanced deflection.

4.4   The foam analogue
Recall that constraining the sand against lateral flow does not significantly affect the response of the beam. Moreover, the impact process in this constrained case as seen in Fig. 10a is very reminiscent of a foam impacting a rigid stationary target. Pingle et al. (2011) developed an analogy between a constrained sand slug and a foam projectile with all properties of the foam derived from the properties of the sand particle. Here, we investigate whether such a foam model can also be used to predict the response of beams impacted by sand slugs.

Plane strain finite deformation FE calculations of foam projectiles impacting beams were conducted using the commercial FE package ABAQUS. Similar to the calculations described above, only half the beams were modelled in the calculations with symmetry imposed at $x_1 = 0$ and all degrees of freedom constrained at $x_1 = L$ to simulate the clamped boundary conditions. The beam and the foam projectile were discretised using 4-noded plane strain elements with reduced integration (CPE4R in

the ABAQUS notation). Typically, the elements were square and had a size $h/15$ in the beam and $H/100$ in the foam projectile. The projectile was given an initial velocity v_o and brought into contact with the beam at mid-span at time $t = 0$ with contact between the beam and projectile modelled using the general contact option in ABAQUS. The beam material was modelled as elastic-plastic using J2 flow theory and properties as specified in Section 3.1. The properties endowed to the foam projectile are those specified by Pingle et al. (2011) and detailed below.

The foam of initial density $\bar{\rho}\rho_s$ is modelled as a compressible continuum using the metal foam constitutive model of Deshpande and Fleck (2000). Write s_{ij} as the usual deviatoric stress and the von Mises effective stress as $\sigma_e \equiv \sqrt{3s_{ij}s_{ij}/2}$. Then, the isotropic yield surface for the metal foam is specified by

$$\hat{\sigma} - Y = 0, \tag{4.7}$$

where the equivalent stress $\hat{\sigma}$ is a homogeneous function of σ_e and mean stress $\sigma_m \equiv \sigma_{kk}/3$ according to

$$\hat{\sigma}^2 \equiv \frac{1}{1+(\alpha/3)^2}\left[\sigma_e^2 + \alpha^2\sigma_m^2\right]. \tag{4.8}$$

The material parameter α denotes the ratio of deviatoric strength to hydrostatic strength, and the normalisation factor on the right hand side of relation (4.8) is chosen such that $\hat{\sigma}$ denotes the stress in a uniaxial tension or compression test. An over-stress model is employed with the yield stress Y specified by

$$Y = \eta\dot{\hat{\varepsilon}}^p + \sigma_c, \tag{4.9}$$

in terms of the viscosity η and the plastic strain-rate $\dot{\hat{\varepsilon}}^p$ (work conjugate to $\hat{\sigma}$). The characteristic $\sigma_c(\hat{\varepsilon}^p)$ is the static uniaxial stress versus plastic strain relation of the foam. Normality of plastic flow is assumed, and this implies that the "plastic Poisson's ratio" $\nu_p = -\dot{\varepsilon}_{22}^p/\dot{\varepsilon}_{11}^p$ for uniaxial compression in the 1-direction is given by

$$\nu_p = \frac{1/2 - (\alpha/3)^2}{1+(\alpha/3)^2}. \tag{4.10}$$

The modulus of the foam is related to the sand particle properties via the relation

$$E_f = \frac{3\sqrt{3}K_n}{4b} \tag{4.11}$$

and both elastic Poisson's ratio and a plastic Poisson's ratio $\nu_p = \nu = 0$. Since the foam is used to model a loose aggregate of particles which have no static strength up to densification we assume a static strength of the form

$$\sigma_c = \begin{cases} \sigma_{pl} & \hat{\varepsilon}^p \le -\ln(1 - \varepsilon_D) \\ \infty & \text{otherwise} \end{cases} \tag{4.12}$$

where ε_D is the nominal densification strain given by

$$\varepsilon_D = 1 - \frac{\bar{\rho}}{\bar{\rho}_{\max}}. \tag{4.13}$$

and $\sigma_{pl} = \bar{\rho}\rho_s v_o^2/100$ (i.e. significantly less than the dynamic strength but sufficient to give numerical stability). The discrete calculations suggest that $\bar{\rho}_{\max} = 0.9$ and this

is the value used in Eq. (4.13). It now remains to specify the material viscosity η.
Radford et al. (2005) have shown that the linear viscosity η results in a shock within
the foam of width l given by

$$l = \frac{\eta \varepsilon_D}{\bar{\rho} \rho_s v_o} \tag{4.14}$$

Using (4.14), we choose a viscosity so that $w \ll H$ in all the calculations consistent
with the discrete calculations of Pingle et al. (2011) that suggested that the shock
width was no larger than about 10 particle diameters.

Comparisons between the full discrete calculations and the predictions using the foam
projectile are shown in Fig. 4. The foam projectile loading captures the deformation
of the beam with remarkable accuracy. The slight over-prediction of the deflection is
due to the fact that the foam projectile does not capture the reduction in the applied
pressure due to the lateral spreading of the sand. We conclude that the foam projectile
represents a good compromise between accuracy and computational cost to simulate
the loading of beams by sand slugs.

5.   Impact against sandwich beams

We proceed to analyse the response of sandwich beams subjected to the loading by a
high velocity sand slug.   These sandwich beams have the same areal mass
$m_b = 400 \,\mathrm{kgm}^{-2}$ and half-span $L = 0.5\,\mathrm{m}$ as the monolithic beams analysed in
Section 4.   Also recall that we partition the mass of the sandwich beam as per
Eq. (3.2) so that a third of the mass is in the core and each of the face sheets. These
constraints imply that the deflection w of the back face of the sandwich beam at mid-
span may be written in non-dimensional terms as

$$\bar{w} \equiv \frac{w}{L} = f\left(\bar{t}, \frac{\rho_m}{\rho_s}, \bar{c}, \bar{\sigma}_c, \bar{a}, \bar{I}_o, \bar{m}, \bar{\rho} \right). \tag{5.1}$$

where $\bar{c} \equiv c/L$ is the aspect ratio of the beam and $\bar{\sigma}_c \equiv \sigma_c / (\bar{\rho}_c \sigma_Y)$ is the normalised
core strength: for a given topology, a "stretching-dominated" cellular foam core will
have a unique value of $\bar{\sigma}_c$ independent of its density (Wadley et al., 2003). Note that
the the constraint, Eq. (3.2), implies that the core relative density is not an
independent parameter, but is related to the beam aspect ratio via the relation

$$\bar{\rho}_c \equiv \frac{\rho_c}{\rho_m} = \frac{m_b}{3\rho_m L \bar{c}}. \tag{5.2}$$

The effect of the loading parameters $(\bar{a}, \bar{I}_o, \bar{m}, \bar{\rho})$ was clarified in Section 4. Our aim
here is to contrast the monolithic and sandwich beam responses and hence we focus
attention on the sandwich beam parameter $\bar{\sigma}_c$ and present results for two beam aspect
ratios $\bar{c} = 0.11$ and 0.33 corresponding to core relative densities $\bar{\rho}_c = 0.3$ and 0.1,
respectively. Unless otherwise specified the following reference parameters will be
employed in all the calculations reported here: $\bar{a} = 0.2$, $\bar{\rho} = 0.2$, $\bar{m} = 0.405$ and
$\bar{I}_o = 0.67$ corresponding to a impact velocity $v_o = 400\,\mathrm{ms}^{-1}$. The sandwich parameters
$\bar{c}$ and $\bar{\sigma}_c$ are varied in this study and will be specified in all the calculations reported
below.

The simulated temporal variation of the normalised deflection $\bar{w}$ is shown in Figs. 11a and 11b for sandwich beams with aspect ratios $\bar{c} = 0.11$ and 0.33 respectively, and selected values of the core strength $\bar{\sigma}_c$. The response of the equivalent monolithic beam is also included in Fig. 11. The deflections typically decrease with increasing $\bar{\sigma}_c$ for both values of $\bar{c}$. However, there is a key difference between the two sandwich beam aspect ratios. While the peak deflections w_{max} of the $\bar{c} = 0.11$ sandwich beams with weak cores (i.e. low $\bar{\sigma}_c$ values) are comparable to their monolithic counterparts, the $\bar{c} = 0.33$ sandwich beams have a significantly lower peak deflections compared to the equal mass monolithic beams over the entire range of $\bar{\sigma}_c$ considered here.

Snapshots from the simulations showing the deformation of the sand slug and the $\bar{c} = 0.11$ and 0.33 sandwich beams ($\bar{\sigma}_c = 0.11$) are shown in Figs. 12a and 12b, respectively. The overall deformation of the sand slug is similar to that seen in Fig. 5 for the monolithic beams. This process comprises first the transmission of a densification front through the slug, then the lateral spreading of the sand and followed by loss of contact between the beam and the sand as the beam bounces back after attaining its peak deflection. However, the deformation of the sandwich beams is markedly different from the monolithic beams. In the initial phase, the deformation comprises mainly of core compression with negligible deflection of the back face of the sandwich beam. This is followed by the bending of the entire beam. For the cases shown in Figs. 12a and 12b, the core compression phase typically ends before the peak deflection has been attained.

Predictions of the core compression strain $\varepsilon_c \equiv \Delta c / c$, where Δc is the reduction in core thickness at the mid-span are included in Figs. 13a and 13b for the $\bar{c} = 0.11$ and $\bar{c} = 0.33$ cases, respectively (corresponding to the $\bar{w}$ predictions in Fig. 11). The core compression increases with decreasing $\bar{\sigma}_c$ but the time t_c required for the maximum core compression ε_c^{max} to be attained is reasonably independent of $\bar{\sigma}_c$. The effects of core strength $\bar{\sigma}_c$ on the normalised maximum deflections $\bar{w}_{max} \equiv w_{max} / L$, maximum core compression ε_c^{max} and the normalised time $\bar{t}_c \equiv t_c / L\sqrt{\rho_m / \sigma_Y}$, are included in Figs. 14a, 14b and 14c, respectively. Recall that the maximum deflection of the equivalent monolithic beam subjected to the same loading is $\bar{w}_{max} \approx 0.14$. Thus, over the entire range of $\bar{\sigma}_c$ values considered, the $\bar{c} = 0.33$ sandwich beam outperform their monolithic counterparts in terms of having of lower $\bar{w}_{max}$ while the $\bar{c} = 0.11$ sandwich beams have a superior performance only for $\bar{\sigma}_c$ values greater than 0.2. The core compression ε_c^{max} and the corresponding core compression time $\bar{t}_c$ were higher for the $\bar{c} = 0.33$ sandwich beam compared to the $\bar{c} = 0.11$ beam for all values of $\bar{\sigma}_c$ considered here. This observation is rationalised as follows. Recall that the $\bar{c} = 0.33$ core has a relative density $\bar{\rho}_c = 0.1$ while $\bar{\rho}_c = 0.3$ for the $\bar{c} = 0.11$ beam. Thus, the actual core strength σ_c of the $\bar{c} = 0.33$ beam is lower compared to the $\bar{c} = 0.11$ beam for any given values of normalized strength, $\bar{\sigma}_c$. This lower actual

core strength results in a large core compression and a longer core compression time $\bar{t}_c$.

5.1 Analysis of the superior performance of sandwich beams

The superior performance of appropriately designed sandwich beams can in general be a result of two effects:

(i)   A fluid-structure interaction effect whereby the pressure exerted by the sand on the sandwich beam is less than that exerted on the monolithic beam (and consequently the momentum transmitted to the sandwich panel is less than that transmitted into the monolithic beam).   This was the dominant phenomenon responsible for the superior performance of sandwich panels subjected to underwater blast loading (Fleck and Deshpande, 2004).

(ii)   The structural performance of sandwich beams is superior to monolithic beams.

In order to understand the relative contributions from these two effects we first plot in Fig. 15 the temporal variation of the normalised pressure $\bar{p}/(\bar{\rho}\rho_s v_o^2)$ for the $\bar{c} = 0.33$ sandwich beam and two extreme values of $\bar{\sigma}_c$ ($\bar{p}$ is calculated in the manner described in Section 4 for the monolithic beams).   The corresponding prediction for the monolithic beam is also included in Fig. 15.   It is clear that the loading due to the sand is nearly identical for the sandwich and monolithic beams and thus the superior performance of the sandwich beams must be due to differences in the structural performance of the sandwich and monolithic beams.   We discuss the two factors that result in difference between the dynamic performance of sandwich and monolithic beams.

(a)   *The sandwich effect*:   The static bending strength of sandwich beams is typically greater than that of monolithic beams of equal mass.   Thus, as long as the deflection of the sandwich beams is less than the beam thickness (i.e. response is dominated by the bending strength rather than stretch resistance), the sandwich beams deflect less than monolithic beams for a given applied load.   If the load is sufficiently large that the deflections are greater than the beam thickness, the response becomes dominated by beam stretching and sandwich beams then loose their advantage over monolithic beams.   It is clear from Fig. 14a that $w_{\max}/c < 1$ for the $\bar{c} = 0.33$ beams over the entire range of $\bar{\sigma}_c$ considered, i.e. the beams are in a regime where sandwich beams are expected to outperform monolithic beams.   On the other hand, the smaller core thickness of the $\bar{c} = 0.11$ beams implies that for the loadings considered here $w_{\max}$ typically exceeds c and these beams thus have a performance comparable to their monolithic counterparts.   We note in passing that core compression reduces with increasing $\bar{\sigma}_c$.   This means that the separation of the two face sheets and the sandwich effect is maintained for high core strengths resulting in a reduction in $w_{\max}$ with increasing $\bar{\sigma}_c$.

(b)   *Coupling between the core compression and beam bending phases*: McShane et al. (2007) and Liang et al. (2007) identified the sol-called soft-core effect in which the sandwich beams outperformed monolithic beams under dynamic loading conditions when there was a strong temporal coupling between the core compression and beam bending phases of the deformation, i.e. the core compression time t_c was comparable to the time $t_{\max}$ required for the beams to attain their maximum deflection.

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Predictions of $\bar{t}_c$ included in Fig. 14c indicate that while there is significant coupling of the core compression and beam bending phases at low values of $\bar{\sigma}_c$, the two phases are decoupled at the high values of $\bar{\sigma}_c$, i.e. in the range where the performance benefits of using sandwich construction are maximum the core compression and beam bending phases are decoupled. We thus conclude that sandwich effect is the largest contributor to the performance benefits observed in the sandwich simulations reported here. These performance benefits are the maximum for high values of $\bar{c}$ and occur over a wide range of sand slug impulses as seen in Fig. 16 for the $\bar{c} = 0.33$ sandwich beams.

5.2   Foam analogue

The foam projectile was shown to accurately represent sand slug loading and capture the deformation of monolithic beams subjected to sand slug loading. Here we demonstrate that this foam projectile analogy also works for sandwich beams loaded by sand slugs.

The properties of the foam projectile as chosen as described in Section 4.4 and calculations of foam projectile loading of the sandwich beams carried out in the commercial FE package ABAQUS. Predictions of $\bar{w}_{\max}$, $\varepsilon_c^{\max}$ and $t_c / t_{\max}$ of the $\bar{c} = 0.11$ and 0.33 sandwich beams are included in Fig. 14 while predictions of $\bar{w}_{\max}$ for the $\bar{c} = 0.33$ sandwich beam are included in Fig. 16 as a function of $\bar{I}_o$ for two selected values of $\bar{\sigma}_c$ (labelled foam impact in these figures). In all cases excellent agreement is obtained between the predictions of the of the sand slug impact and the foam projectile equivalent. We thus conclude that foam projectiles can also be used to simulate sand slug loading of sandwich beams.

6.   Circular plates loaded by cylindrical sand slugs

We proceed to briefly illustrate that the main mechanisms described above are not restricted to beams impacted by sand slug comprising cylindrical sand particles but also hold for circular plates impacted by sand slugs comprising spherical sand particles. Here we analyse the response of circular monolithic and sandwich plates (Fig. 17) impacted centrally and normally by a cylindrical sand slug of radius a and height H, comprising spherical sand particles of diameter D and with a spatially uniform relative density $\bar{\rho}$. Before impact, the sand particles all have a velocity v_o in the negative z direction as shown in Fig. 17. The sand slug impacts clamped circular plates of radius R and areal mass m_b at time $t = 0$.

6.1   Pseudo three-dimensional calculations

The boundary value problems sketched in Fig. 17 are inherently three-dimensional (3D) as we need to model spherical sand particles. However, in order to reduce the computational cost, we analysed these problems using a pseudo 3D technique wherein the sand slug is modelled in a 3D setting but the plate is assumed to undergo axi-symmetric deformation. We briefly explain this methodology.

A slice of the cylindrical sand slug subtending an angle ϕ about the central axis as shown in Fig. 18 was modelled using the GRANULAR package of the discrete code

LAMMPS. The circumferential expansion of this slice was constrained, i.e. the sand particles were constrained by radial rigid stationary walls along $\theta = 0$ and $\theta = \phi$ as shown in Fig. 18. This ensures that consistent with the plate deformation, the deformation of the cylindrical sand slug is also approximately axi-symmetric. Contact between the spherical sand particles of diameter D was characterised by the model described in Section 2.1.

These discrete particle calculations are coupled to the axi-symmetric FE calculation as follows. At any time instant t, the FE calculation provides the radial and axial coordinates (r_p, z_p) and radial and axial velocities (v_r, v_z) of every material point on the plate surface. Recalling that the FE calculation is axi-symmetric the coordinate z_p is independent of θ and the tangential velocity $v_\theta = 0$. The contact forces between the particles and the plate surface (F_r and F_z in the radial and axial directions, respectively) are then calculated as described in Section 2.3 and a vector of nodal forces determined (the force F_θ due to any motion of the sand particles in the θ direction is neglected as it cannot be included in the axi-symmetric FE calculation). These forces are then scaled by the factor $2\pi / \phi$ and inserted into the vector of global nodal forces $\mathbf{F}$ in Eq. (2.5). Subsequently, the discrete and finite element equations are integrated as described in Sections 2.1 and 2.2 and the new positions and velocities of the particles in the discrete calculations and material points in the finite element calculations determined at time $t + \Delta t$.

6.2   Geometry and material properties

Clamped plates of radius $R = 0.5\,\mathrm{m}$ and areal mass $m_b = 160\,\mathrm{kgm}^{-2}$ were analysed. The plates were taken to be made from the same materials as the beams described in Sections 4 and 5, i.e. monolithic plates made from DH36 steel and sandwich plates with DH36 steel face sheets and a foam core also made from DH36 parent material. The monolithic plates had a thickness $2h = 20\,\mathrm{mm}$ and the sandwich plates of equal areal mass were designed in a manner similar to the sandwich beams such that the mass was equally distributed between the two face-sheets each of thickness $h_f = 6.67\,\mathrm{mm}$ and core of thickness c that is related to relative density $\bar{\rho}_c$ of the core via Eq. (5.2). Similar to the sandwich beams, we shall present results for two core relative densities $\bar{\rho}_c = 0.1$ and $\bar{\rho}_c = 0.3$ corresponding to $\bar{c} \equiv c / R = 0.044$ and 0.133. The material properties of the monolithic plate material, the sandwich plate face sheets and the foam core are identical to those described in Section 3.1. The only free material parameter is the core strength σ_c: results are presented for a range of values of σ_c in order to parameterise the effect of core strength.

In all the calculations, the sand slug has a radius $a = 100\,\mathrm{mm}$ so that $a / R = 0.2$, initial height $H = 150\,\mathrm{mm}$ with a relative density $\bar{\rho} = 0.2$ of the sand particles of diameter $D = 200\,\mu\mathrm{m}$. The sand particles were assumed to be made from silica with density and contact properties listed in Section 3.2.

We note that the mid-span deflections of the monolithic and sandwich plates made from DH36 steel may be written in non-dimensional form as (for a given set of sand properties)

$$\overline{w} \equiv \frac{w}{R} = f\left(\overline{t}, \frac{h}{R}, \frac{\rho_m}{\rho_s}, \overline{a}, \overline{I}_o, \overline{m}, \overline{\rho}\right) \qquad (6.1)$$

and

$$\overline{w} \equiv \frac{w}{R} = g\left(\overline{t}, \frac{\rho_m}{\rho_s}, \overline{c}, \overline{\sigma}_c, \overline{a}, \overline{I}_o, \overline{m}, \overline{\rho}\right), \qquad (6.2)$$

respectively. The terms are as defined in Sections 4 and 5 with L replaced by R, i.e.

$$\overline{t} \equiv t / \left(R\sqrt{\rho_m / \sigma_Y}\right), \quad \overline{c} \equiv c / R \text{ and } \overline{a} = a / R. \qquad (6.3)$$

Based on the material properties described above, the following non-dimensional groups are fixed in the calculations: $\overline{m} = 0.405$, $h/R = 0.04$, $\overline{a} = 0.2$, $\overline{\rho} = 0.2$ and $\rho_m / \rho_s = 2.96$. Results are presented for the effect of the sand momentum $\overline{I}_o$, sandwich beam aspect ratio $\overline{c}$ and core strength $\overline{\sigma}_c$ on the plate deflections. All calculations reported subsequent were deformed using a cylindrical slice of the sand slug with $\phi = 10^o$: spot calculations demonstrated that increasing the value of ϕ had no appreciable effect on the predictions.

6.3   Response of monolithic and sandwich plates

Predictions of the normalised maximum back face deflections $\overline{w}_{\max} \equiv w_{\max} / R$ at the mid-span of the monolithic and the $\overline{c} = 0.133$ sandwich plates are plotted in Fig. 19a as a function of the normalised sand slug impulse $\overline{I}_o$. Results for the sandwich plate are included for two values of the core strength $\overline{\sigma}_c$. Over the entire range of impulses considered, the sandwich plates have a lower deflection compared to the monolithic plates. Further, similar to the sandwich beams, the deflections are lower for the sandwich plates with the higher core strength. Snap shots showing the deformation of $\phi = 10^o$ slices of the monolithic and sandwich ($\overline{c} = 0.133$, $\overline{\sigma}_c = 0.21$) plates are included in Figs. 20a and 20b, respectively. The deformation modes are qualitatively very similar to the monolithic and sandwich beams; see Figs. 5 and 12, respectively. Thus, we anticipate that all the findings for the beams will qualitatively extrapolate to the plates.

The normalised maximum back face deflections, $\overline{w}_{\max}$, are summarised in Fig. 19b for $\overline{c} = 0.044$ and 0.133 sandwich plates as a function of the core strength $\overline{\sigma}_c$. The corresponding deflection of the monolithic plate of equal area mass is also included. Consistent the findings for the sandwich beams, the sandwich plate with the thicker core (i.e. $\overline{c} = 0.133$) outperforms both the monolithic plate and the $\overline{c} = 0.044$ sandwich plate over the entire range of $\overline{\sigma}_c$ values investigated here. By contrast, the $\overline{c} = 0.044$ sandwich plates have a smaller deflection compared to the monolithic plates only for $\overline{\sigma}_c > 0.2$, i.e. strong cores. Similar to the beams, the superior performance of these sandwich plates is due to the "sandwich effect" discussed in Section 5.1: the sandwich effect is greater for thicker cores and hence the $\overline{c} = 0.133$ plates outperform the thinner $\overline{c} = 0.044$ plates.

6.4   Foam analogue

In Sections 4.4 and 5.2 we demonstrated that the response of monolithic and sandwich beams loaded by sand slugs was adequately modelled by replacing the sand slug by an

equivalent foam projectile. We now extend this analogy to the plates analysed in this section.

Here we replace the cylindrical sand slug by an equivalent cylindrical foam projectile of the same overall dimensions and density as the un-deformed sand slug and travelling with an initial velocity equal to that of the sand particles. The material properties of the foam are identical to those detailed in Section 4.4 with two exceptions. The sand slug now comprises of spherical rather than cylindrical sand particles. These particles can pack to a maximum relative density $\bar{\rho}_{max} = 0.74$ (rather than the $\bar{\rho}_{max} = 0.9$ in the 2D case). Assuming that $\bar{\rho}_{max}$ corresponds to a cubic close packing, the modulus E_f of the foam under uniaxial straining is then determined by using the Octet truss properties given by Deshpande et al. (2001). This analysis gives E_f in terms of the sand properties as

$$E_f = \frac{4\sqrt{2}K_n}{3D} .$$
(6.4)

Thus, the only modifications to the prescription presented in Section 4.4 are: (i) Eq. (4.11) is replaced by Eq. (6.4) and (ii) use $\bar{\rho}_{max} = 0.74$ in Eq. (4.13) rather than $\bar{\rho}_{max} = 0.9$.

Axisymmetric FE calculations of the foam projectiles impacting the monolithic and sandwich plates were performed using ABAQUS. Predictions of $\bar{w}_{max}$ are included in Figs. 19a and 19b and clearly show the excellent agreement between the full discrete calculations and the calculations using the equivalent foam projectiles.

7.   Concluding remarks

We have presented a coupled discrete/continuum computational framework to analyse the loading of monolithic and sandwich structures by sand columns (slugs) travelling at high velocities. The sand is modelled as either discrete identical circular cylindrical or spherical particles while the structures (beams or circular plates) are modelled using a Lagrangian finite element (FE) framework. The clamped beams and plates are loaded by sand slugs impacting the structures normally and centrally: this is a model problem devised to develop a fundamental understanding of the sand-structure interaction that occurs in more complex loading events such as the loading of structures by landmine explosions.

The calculations of the monolithic beams have demonstrated that the loading due to the sand is primarily inertial with only a small fluid-structure interaction effect, i.e. the momentum transmitted to the structures was approximately equal to the momentum of the incoming sand slug and relatively insensitive to the ratio of the areal masses of the sand slug and plate. The loading and the structural response was well characterised by accounting for just two parameters: (i) the initial momentum of the sand slug and (ii) the loading time given by the ratio of the height of the slug and the initial velocity of the sand particles. The effect of parameters such as the areal mass ratio of the sand slug and structure, relative density of the sand particles in the sand slug and beam aspect ratio are all captured within a single non-dimensional parameter, viz, the ratio of the loading time to the structural response time. Sandwich

beams with thick strong cores were shown to significantly outperform monolithic beams of equal areal mass. This enhanced performed of the sandwich beams was due to the "sandwich effect" whereby the high bending strength gives the sandwich beams superior performance. The main sand/structure interactions mechanisms elucidated by the beam calculations are shown to hold in a three-dimensional setting where slugs comprising spherical sand particles impact clamped circular monolithic and sandwich plates. Again, fluid-structure interaction effects were found to be negligible with sandwich plates with thick and strong cores out-performing monolithic plates of equal areal mass.

The impact of a sand slug against the structures is shown to the analogous to a foam impact. We show that replacing the sand slug by an equivalent foam projectile captures the response of the monolithic and sandwich beams and plates with remarkable accuracy. This foam analogy thus not only presents a numerically efficient method to solve the sand-structure interaction problem but also suggests that foam projectile loading might be a convenient physical analogue for studying this interaction in a laboratory setting.

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Figure Captions
Figure 1: Sketch of a prototypical problem of a clamped sandwich structure loaded by a shallow mine explosion.

Figure 2: The plane strain boundary value problem of a slug of sand comprising circular cylindrical particles impacting a clamped (a) monolithic and (b) sandwich beam.

Figure 3: Sketch illustrating the contact law between two particles in the discrete calculations.

Figure 4: (a) The normalised mid-span deflection $\bar{w}$ versus normalised time $\bar{t}$ histories for three selected values of the normalised sand slug impulse $\bar{I}_o$. (b) The corresponding predictions of the normalised maximum mid-span deflection $\bar{w}_{\max}$ as a function of $\bar{I}_o$. All other parameters are held fixed at their reference values. Predictions using the equivalent foam projectile are also included in (a) and (b).

Figure 5: Snapshots from the coupled discrete/continuum calculations of the sand slug impacting the monolithic beam at six selected normalised times $\bar{t}$. The results are shown for a normalised impulse $\bar{I}_o = 0.67$ with all other parameters fixed at their reference values.

Figure 6: The distribution of the normalised pressure $p/(\bar{\rho}\rho_s v_o^2)$ over the span of the beam at three times $\bar{t}$ after impact of the sand slug against the monolithic beam with a normalised impulse $\bar{I}_o = 0.67$ (all other parameters fixed at their reference values). The coordinate X denotes the x_1 coordinate a material point on the beam surface in its undeformed configuration, i.e. $X = 0$ is at the beam mid-span and $X = L$ is at the supports.

Figure 7: Predictions of (a) the normalised average pressure $\bar{p}/(\bar{\rho}\rho_s v_o^2)$ and (b) normalised transmitted momentum I_t/I_o a function of the normalised time $\hat{t} \equiv tv_o/H$ for the sand slug impact against the monolithic beam. Results are shown for two values of the normalised momentum $\bar{I}_o$ (all other parameters fixed at their reference values).

Figure 8: Predictions of the normalised maximum deflection $\bar{w}_{\max}$ of the monolithic beams as a function of the normalised loading time $\bar{\tau}$. Two sets of calculations are reported: (i) vary $\bar{m}$ with $\bar{\rho} = 0.2$ and (ii) vary $\bar{\rho}$ with $\bar{m} = 0.405$. In each case results are presented for two values of the normalised impulse $\bar{I}_o = 0.67$ and 1.0.

Figure 9: (a) Predictions of the normalised maximum deflection $\bar{w}_{\max}$ of the monolithic beam as a function of the normalised width $\bar{a}$ of the sand slug for two selected values of the sand slug impulse $\bar{I}_o$. (b) Snapshots showing the deformation of the monolithic beam and sand slug at six selected times $\bar{t}$ for the $\bar{a} = 0.8$ slug with an impulse $\bar{I}_o = 0.67$. All parameters not listed here have their reference values.

Figure 10:   (a) Snapshots showing the deformation of the monolithic beam and *constrained* sand slug with an impulse $\bar{I}_o = 0.67$ at six selected times $\bar{t}$.   (b) Predictions of the normalised maximum deflections $\bar{w}_{\max}$ as a function of the sand slug impulse $\bar{I}_o$.   Results are included for two relative densities $\bar{\rho}$ of both the constrained and unconstrained sand slugs. All parameters not listed here have their reference values.

Figure 11: Predictions of the normalised deflection $\bar{w}$ versus normalised time $\bar{t}$ histories of the (a) $\bar{c} = 0.11$ and (b) $\bar{c} = 0.33$ sandwich beams impacted by sand slugs with a normalised impulse $\bar{I}_o = 0.67$.   Results are shown for three values of the normalised core strength $\bar{\sigma}_c$ of the sandwich beams. Predictions of the deflection of the monolithic beam of equal areal mass are also included. All parameters not listed here have their reference values.

Figure 12:   Snapshots at six selected times $\bar{t}$ showing the deformation of the (a) $\bar{c} = 0.11$ and (b) $\bar{c} = 0.33$ sandwich beams with core strength $\bar{\sigma}_c = 0.11$ impacted by sand slugs with a normalised impulse $\bar{I}_o = 0.67$. All parameters not listed here have their reference values.

Figure 13:   Predictions of the variation of the core compression ε_c with normalised time $\bar{t}$ for the (a) $\bar{c} = 0.11$ and (b) $\bar{c} = 0.33$ sandwich beams impacted by sand slugs with a normalised impulse $\bar{I}_o = 0.67$. Results are shown for the three selected values of core strength $\bar{\sigma}_c$ corresponding to Fig. 11 with all parameters fixed at their reference values.

Figure 14:   Predictions of (a) normalised maximum deflection $\bar{w}_{\max}$, (b) maximum core compression $\varepsilon_c^{\max}$ and (c) normalised core compression time $\bar{t}_c$ as a function of the normalised core strength $\bar{\sigma}_c$ for the $\bar{c} = 0.11$ and $\bar{c} = 0.33$ sandwich beams impacted by sand slugs with a normalised impulse $\bar{I}_o = 0.67$. All parameters not listed here have their reference values. Predictions using the equivalent foam projectile are included in (a) and (b).

Figure 15: Predicted normalised average pressure $\bar{p} / (\bar{\rho} \rho_s v_o^2)$ versus normalised time $\bar{t}$ history for the $\bar{c} = 0.33$ sandwich beams impacted by the sand slug with impulse $\bar{I}_o = 0.67$. Results are shown for two extreme values of the normalised core strength $\bar{\sigma}_c$ with all other parameters equal to their reference values. The corresponding predictions for the equal areal mass monolithic beam are also included.

Figure 16: Predictions of the normalised maximum mid-span deflection of the back face of the $\bar{c} = 0.33$ sandwich beams as a function of the sand slug normalised impulse $\bar{I}_o$. Results are shown for two values of the normalised core strength $\bar{\sigma}_c$ with all other parameters equal to their reference values. Predictions using the equivalent foam projectile are also included.

Figure 17: Sketches illustrating the geometries of the clamped circular (a) monolithic and (b) sandwich plates impacted by a cylindrical sand slug comprising identical spherical sand particles.

Figure 18: Sketch illustrating the slice of the circular cylindrical sand slug analysed using discrete calculations. The applied boundary conditions on the sides of the slice are also illustrated.

Figure 19: (a) Predictions of the normalised maximum deflection $\bar{w}_{\max}$ of the monolithic and the $\bar{c} = 0.133$ sandwich plate as a function of the normalised sand slug impulse $\bar{I}_o$. Results for the sandwich plate are shown for two values of the normalised core strength $\bar{\sigma}_c$. (b) The deflections $\bar{w}_{\max}$ of the $\bar{c} = 0.044$ and $\bar{c} = 0.133$ sandwich plates as a function of $\bar{\sigma}_c$ for an impulse $\bar{I}_o = 0.67$. The corresponding deflection of the monolithic plate of equal areal mass is included in (b). Predictions using the equivalent foam projectile are also included in (a) and (b).

Figure 20: Snapshots at six selected times $\bar{t}$ showing the deformation of the (a) monolithic and (b) $\bar{c} = 0.133$ sandwich plate with core strength $\bar{\sigma}_c = 0.21$ impacted by sand slugs with a normalised impulse $\bar{I}_o = 0.67$. All parameters not listed here have their reference values. The figures show the $\phi = 10^o$ slice analysed in the discrete calculations.