

# FATIGUE CRACK MEASUREMENT: TECHNIQUES AND APPLICATIONS

Editors

K.J. MARSH, R.A. SMITH and  
R.O. RITCHIE

(1991)

**EMAS**  
■■■■□■

Engineering Materials  
Advisory Services Ltd.  
339, Halesowen Road, Cradley Heath,  
Warley, West Midlands B64 6PH, UK

## 4 COMPLIANCE METHODS FOR MEASUREMENT OF CRACK LENGTH

N.A. Fleck

### 4.1 INTRODUCTION

The presence of a crack in a structure leads to an increase in the compliance of that structure. The change in compliance manifests itself in a number of ways. It leads to a decrease in resonant frequencies, and it leads to load redistribution in a statically indeterminate structure. Measurement of the compliance of a cracked structure is a useful means of measuring the crack length. It is also a simple and accurate method for determining the crack closure response of a specimen. In many cases, both the crack length and the closure level are needed to quantify sub-critical crack growth.

The compliance of a structure is closely related to the crack tip fracture mechanics parameters, as follows. Consider a cracked structure under remote load  $P$  such that the loading points displace by a distance  $v$ . The compliance of the structure  $C$  is given by  $C=v/P$  and is dependent upon the crack length. The crack tip driving force, called the strain energy release rate  $G$ , is related to the change in compliance with crack length by,

$$G = \frac{1}{2} P^2 \frac{dC}{da} \quad (1)$$

The extra compliance of a structure is the additional compliance due to the presence of a crack. The ease with which we can measure this extra compliance is very sensitive to the location of the displacement gauge. This may be demonstrated by considering the displacement field in an infinite plate under a remote tensile stress  $\sigma$ , containing a central crack of length  $2a$ , see Figure 4.1. The plate is linear elastic in response with a

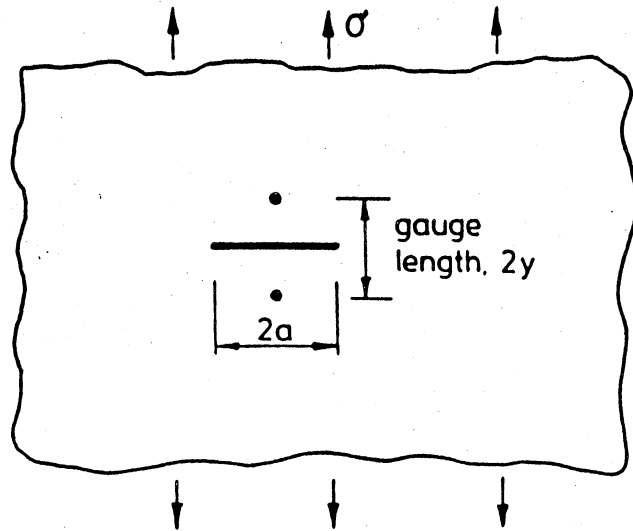


FIGURE 4.1 Location of displacement gauge on centre cracked panel under remote tensile stress  $\sigma$ .

Young's modulus  $E$  and a Poisson's ratio  $\nu$ . Assume the displacement gauge straddles the crack along the centre line of the structure and has a gauge length of  $2y$ . If no crack is present the total measured displacement is  $2v_0 = 2y\sigma/E$ . We can use complex variable methods[1] in order to determine the extra displacement  $2\Delta v$  due to the presence of the crack. For  $y/a \gg 1$  the extra displacement is given by

$$\frac{\Delta v}{v_0} \approx \frac{3 + \nu}{2} \left(\frac{a}{y}\right)^2 \quad (2)$$

For example, assume  $y=10a$ . Equation (2) gives  $\Delta v/v_0 = 0.017$  for  $\nu = 0.3$ . For the case of a penny shaped crack of radius  $a$  in an infinite solid  $\Delta v/v_0$  falls off more rapidly still as  $(a/y)^3$  for  $y/a \gg 1$ . We conclude that the extra displacement due to the presence of the crack falls off rapidly with increasing distance from the displacement gauge to the crack. In order to measure crack length accurately, the displacement gauge should be placed as close to the crack as possible.

In this chapter the use of a variety of compliance gauges is described for measurement of crack length and crack closure response. We discuss the phenomenon of crack closure and explain the techniques used for defining the crack closure load in an unambiguous repeatable fashion. A case study is reported, whereby the crack closure response of BS4360 50B steel is given using several types of compliance gauge.

## 4.2 CLASSIFICATION OF COMPLIANCE GAUGES

Compliance gauges fall into two main classes: gauges which are placed remotely from the crack tip and gauges placed close to the crack tip. Remote gauges are used to measure crack length and to monitor the bulk crack closure response. Near tip gauges are sensitive to the change in specimen compliance associated with crack closure but are of limited use for determining absolute crack length as calibration becomes sensitive to gauge location.

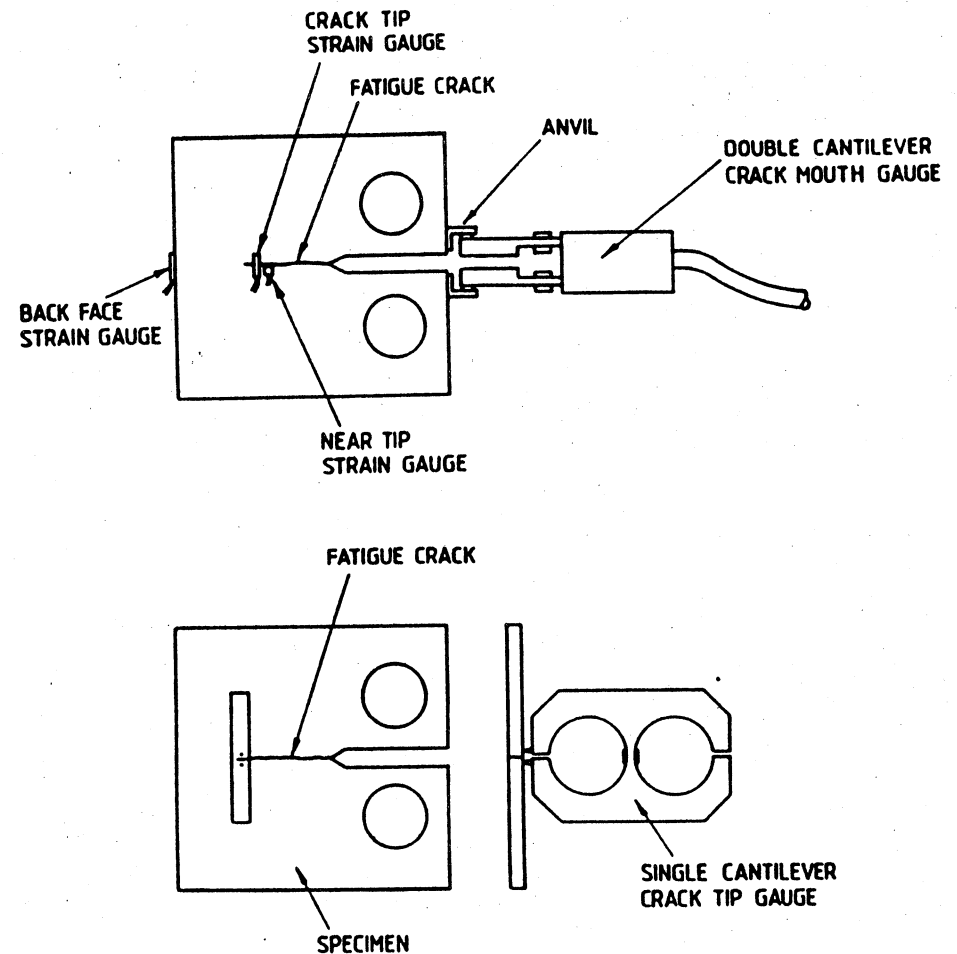


FIGURE 4.2 Common compliance techniques for measuring crack closure.

## 4.2.1 Remote gauges

Commonly used remote gauges are the crack mouth gauge and the back face strain gauge. Both gauges are cheap and simple to use.

- (i) *The crack mouth gauge* clips across the notch mouth of a test-piece such as the compact tension (CT) specimen, or across the crack mid-point in a centre cracked panel. The gauge is made from a single cantilever or from a double cantilever, and displacement is measured via strain gauges on the arms of the gauge, see Figure 4.2. Design guidelines for both types of gauge have been given previously,[2]. In general, the single cantilever gauge is more sensitive, but suffers from the drawbacks that it is delicate and difficult to mount on to a specimen. Usually, the single cantilever gauge is held in place by tension springs or by rubber bands. Sullivan and Crooker[3] have measured experimentally the crack opening displacement  $v$  at the mouth of a compact tension specimen as a function of crack length,  $a$ . A quadratic regression analysis of their data may be used to give a calibration function for this geometry,

$$\frac{vEB}{P} = 65.351 - 298.06 \frac{a}{w} + 630.11 \left(\frac{a}{w}\right)^2, \quad 0.3 \leq \frac{a}{w} \leq 0.6 \quad (3)$$

where  $E$  is Young's modulus,  $B$  is specimen thickness,  $P$  is load, and  $w$  is the width of the specimen.

- (ii) *The back face strain gauge* is useful for bend specimens such as the four point bend specimen, and the compact tension specimen. For the compact tension specimen the back face strain gauge shows less hysteresis, is more sensitive and is less influenced by loading pin mechanical noise than the crack mouth gauge. The back face strain gauge is also effective for tension specimens such as the centre cracked panel.

Richards and Deans[4] have investigated experimentally the relation between back face strain  $\epsilon$  and specimen aspect ratio  $a/w$  for the compact tension specimen. A quadratic regression analysis may be used to express their results in closed form,

$$\frac{\epsilon EBw}{P} = 13.841 - 72.506 \frac{a}{w} + 138.45 \left(\frac{a}{w}\right)^2, \quad 0.3 \leq \frac{a}{w} \leq 0.6 \quad (4)$$

Equations (3) and (4) may be used to measure crack length from specimen compliance. The usual technique in fatigue is to measure the unloading compliance over 10% of the full load range from maximum load of the fatigue cycle. This procedure minimises the effects of crack closure and of crack tip plasticity upon the measured specimen compliance.

## 4.2.2 Near tip gauges

Several types of compliance gauge have been developed for measuring specimen compliance near to the crack tip. Such a positioning gives high sensitivity of the gauge to a change of crack length. The near tip displacement gauge and near tip strain gauge are mounted on the side face of the specimen. Measurement of specimen compliance at the centre of a specimen is more difficult. A push-rod gauge has been developed successfully for this purpose[5]. Each of the gauges will now be described in turn.

- (i) *Displacement gauge.* A single or double cantilever beam displacement gauge is fastened just behind the crack tip, on a side face of the specimen; the arms of the gauge straddle the crack. Fastening of the gauge on the specimen is not trivial. If the gauge has needle tips fastened to each arm, then small Vickers indents may be placed in the specimen to aid location of the gauge. Alternatively, the gauge may be mounted on the test-piece with the aid of small anvils glued to the specimen before commencement of the test. The clip gauge is located on to the anvils by universal ball joints, comprising a 1mm hole drilled in each anvil and a mating 1.6mm diameter steel ball glued to each arm of the clip gauge.

- (ii) *Strain gauge.* A near tip strain gauge either straddles the fatigue crack, or is positioned adjacent to one flank of the crack so that it detects compressive strains when the fatigue crack closes. Both types of gauge are shown in Figure 4.2. Use of the strain gauge straddling the crack is as follows. On interruption of the fatigue test, a 6mm x 1mm strain gauge is glued to the specimen, with the centre line of the gauge 1mm behind the crack tip. The use of 4mm wide cellulose tape between the strain gauge and the specimen ensures that only the ends of the strain gauge are fastened to the specimen. The cellulose tape prevents any adhesive from entering the fatigue crack, and ensures that the strain gauge behaves as a miniature, very sensitive extensometer. The strains in the strain gauge are sufficiently large that the gauge suffers fatigue failure after a few millimetres of crack growth subsequent to its application.

Alternatively, a 1mm x 1mm strain gauge is laid behind the crack tip such that the top of the gauge is aligned with the lower flank of the fatigue crack. As the fatigue crack closes near its tip, the gauge experiences compressive strains. This type of gauge is only successful when it is placed less than approximately 3mm from the crack tip[6].

The near tip strain gauges suffer from the disadvantage that the gauges can only be fastened to a specimen upon interruption of the

fatigue test. Removal of the specimen from the machine is necessary in order to glue the gauge accurately and positively to the specimen.

- (iii) *Push rod gauge.* A push rod gauge may be used to monitor crack opening displacements just behind the crack tip at the centre of a compact tension specimen, Figure 4.3. In order to use the gauge, the fatigue test is interrupted and two parallel holes of diameter 1.5mm are drilled just behind the fatigue crack front. One hole is drilled to a depth of 1mm below the fracture plane, the other to a depth of 1mm

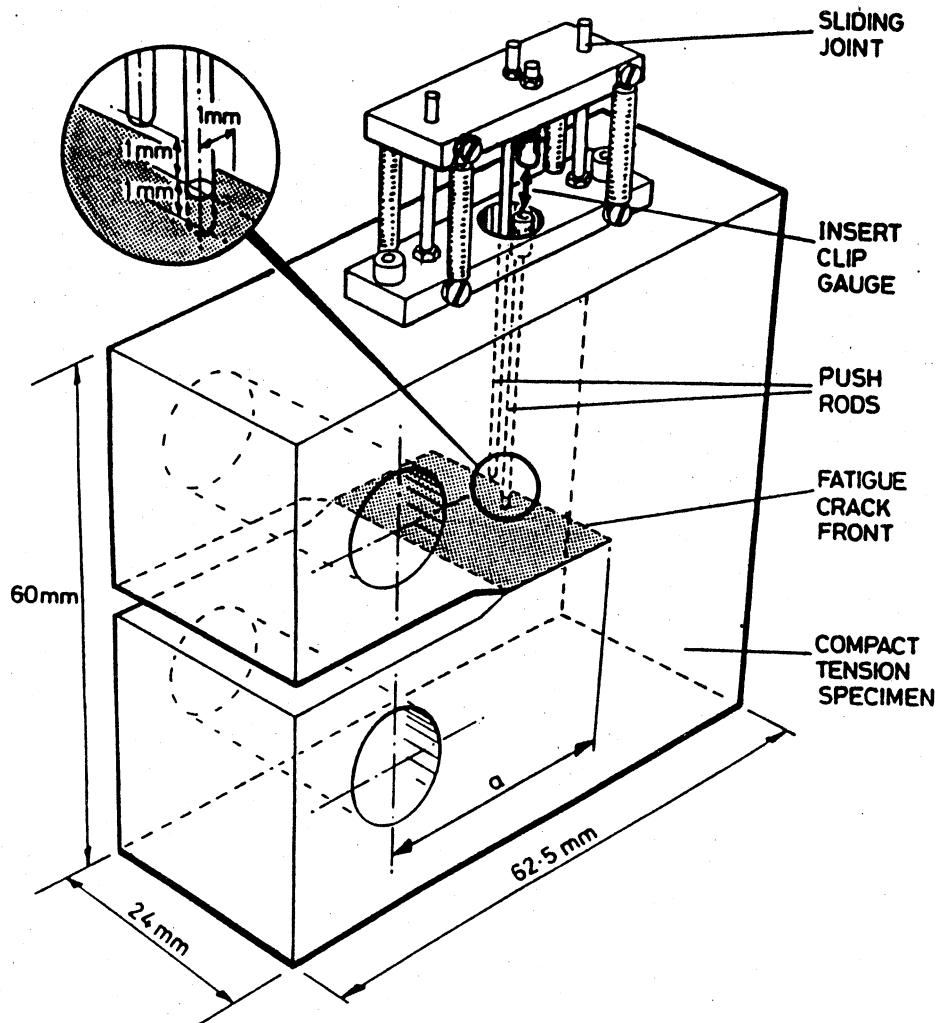


FIGURE 4.3 The push rod closure gauge.

above the fracture plane. A push rod assembly is then fastened to the specimen. When the fatigue test is recommenced, the relative displacement of the hole bottoms is measured with a twin cantilever displacement gauge, via the push rods. As usual, the crack closure load is deduced by locating the point at which the load-displacement trace becomes non-linear.

### 4.3 MEASUREMENT OF FATIGUE CRACK CLOSURE

A crack in an ideal elastic solid opens and closes at zero load. In reality fatigue cracks close above zero load, due to extra material wedged between the crack flanks. Three common sources of crack closure have been identified: plasticity-induced crack closure, roughness-induced crack closure and oxide-induced crack closure.

The stress intensity factor at which a fatigue crack opens  $K_{op}$  is determined by the sum of the contributions from plasticity-induced, roughness-induced and oxide-induced crack closure. Usually it is assumed that no further cyclic straining of material occurs at the crack tip for loads less than the crack opening load  $P_{op}$ . The effective stress intensity range for fatigue crack growth  $\Delta K_{eff}$  is then defined as  $\Delta K_{eff} = K_{max} - K_{op}$ , where  $K_{max}$  is the maximum stress intensity factor of the fatigue cycle. The fraction,  $U$ , of the load cycle for which the crack is open is,

$$U = \frac{K_{max} - K_{op}}{K_{max} - K_{min}} = \frac{\Delta K_{eff}}{\Delta K} \quad (5)$$

Compliance methods have become the standard technique for measuring the crack closure load, although electrical resistance and ultrasonic methods are also used. The compliance techniques detect the decrease in compliance of a cracked specimen associated with closure of the crack, Figure 4.4. In practice, fatigue cracks open progressively to the crack tip and close progressively from the crack tip, making it difficult to locate precisely the load at which the crack tip opens. Plastic deformation near the crack tip complicates the load versus displacement trace by giving rise to hysteresis of the displacement signal.

Potential drop and ultrasonic methods rely upon a decrease in electrical and acoustic resistance of a specimen when the fatigue crack closes. A plot of load versus transmitted signal is used to determine the crack closure load, see Figure 4.5.

Compliance methods give more reliable and consistent measurements of the crack closure response than the electrical or acoustic methods. For example, a fatigue crack may be closed mechanically but open

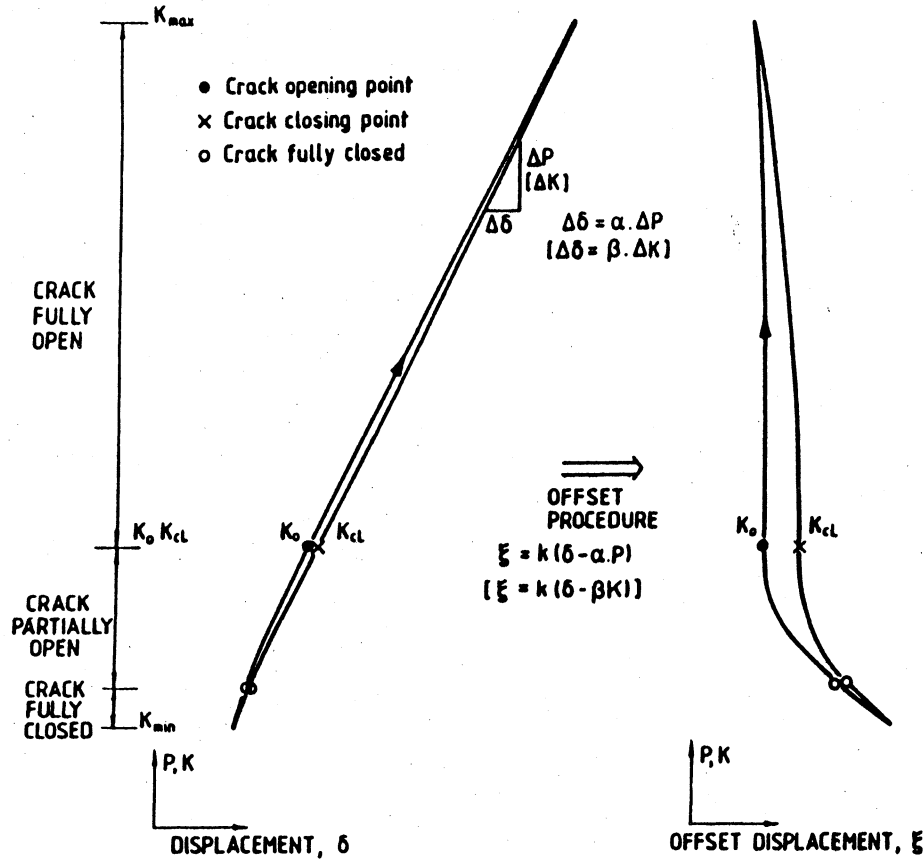


FIGURE 4.4 Effect of closure of fatigue crack upon specimen compliance.

electrically. The surfaces of a fatigue crack oxidise readily, resulting in a tenacious and insulating oxide layer. No indication of the load at which a crack opens or closes may be detected, due to the oxide preventing metal to metal contact. Alternatively, when tested in a vacuum, rough fracture surfaces may touch and slide past each other, yet carry no load. Then, the crack is open mechanically yet closed electrically and the DC potential drop procedure indicates erroneously high crack closure loads. Further sources of error arise from the change in electrical resistance of the specimen associated with crack tip plasticity, and the change of resistance due to intermittent electrical contact between specimen and loading fixture. We conclude that potential drop methods should be used with extreme caution in order to measure the crack closure response. There are several examples in the literature where potential drop techniques

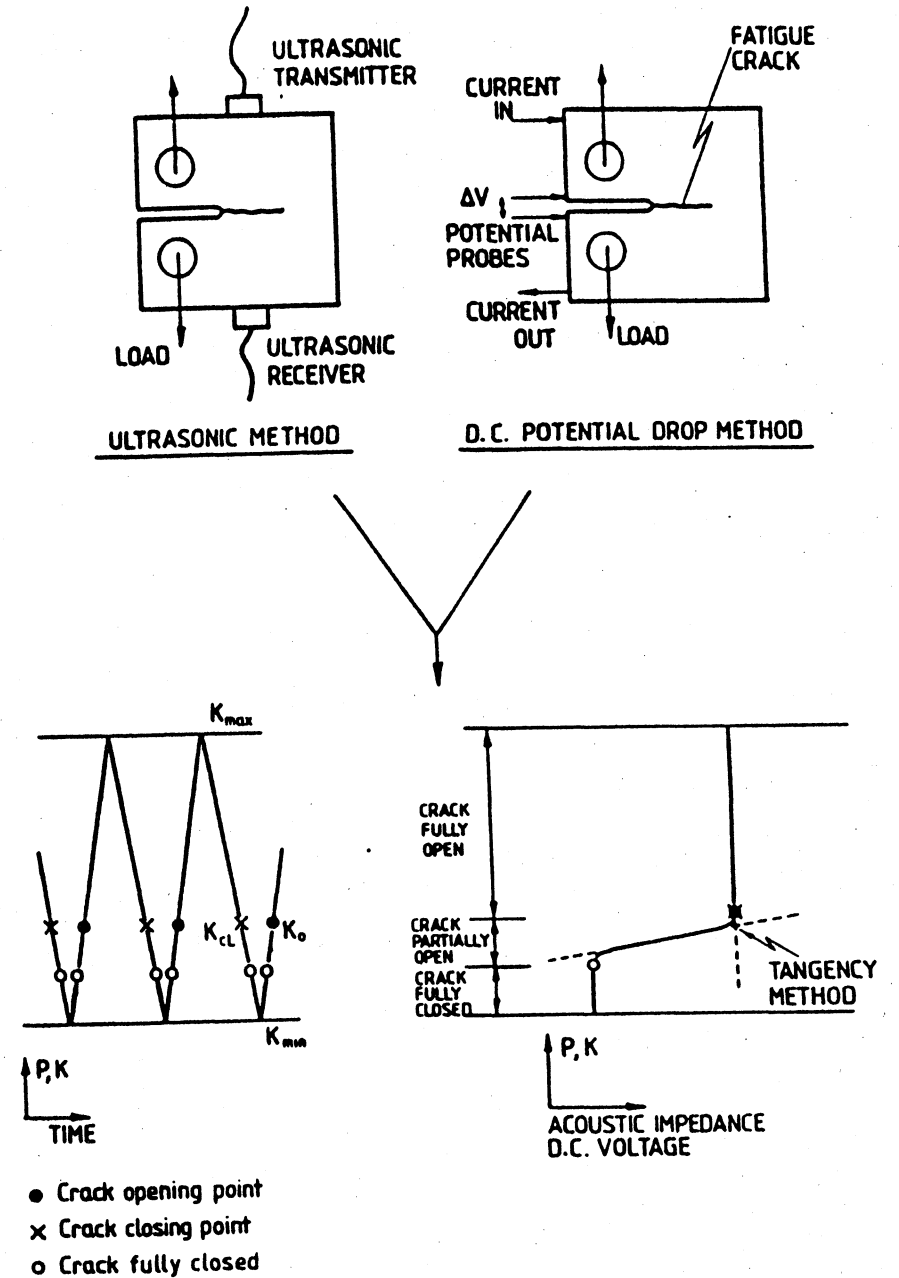


FIGURE 4.5 Effect of closure of fatigue crack upon electrical and acoustic impedance of specimen.

Symbol	Closure Measurement Technique	Load Ratio, R	Thickness, R (mm)	Investigation
○	Potential drop	0.05	12.7	Unangst et al. [7]
●	Potential drop	0.05	25.4	Unangst et al. [7]
□	Compliance	0.08	25.3	Katcher & Kaplan [8]
△	Ultrasonics	0.08	12.5	Frandsen et al. [9]
▲	Ultrasonics	0	25.4	Mahulikar et al. [10]
▽	Ultrasonics	0.06	25.4	Mahulikar et al. [11]
▼	Ultrasonics	0	25.4	Sewell & Marcus [12]

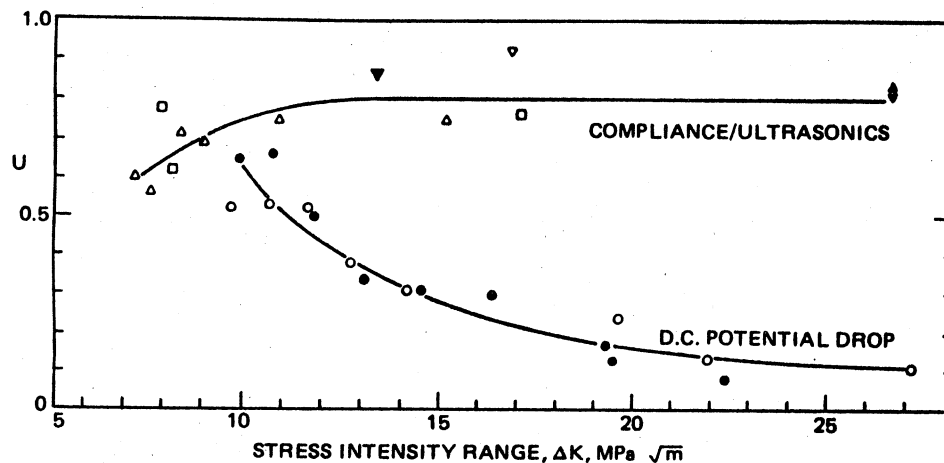


FIGURE 4.6 Comparison of closure results for 2219-T851 aluminium alloy, using compliance, D.C. potential drop and ultrasonic techniques.

indicate a different crack closure behaviour from that inferred by compliance methods. This is demonstrated in Figure 4.6 for the case of 2219-T851 aluminium alloy. Caution should also be exercised in the interpretation of ultrasonic measurements of crack closure. Occasional asperity-asperity contact of mating fatigue fracture surfaces may transfer little load but may transmit an acoustic wave: the crack is open mechanically but closed acoustically. Detailed comparisons of closure measurement techniques have been given by Bachmann and Munz[13,14] and by Fleck[6,15].

#### 4.3.1 Interpretation of load-displacement trace

No single definition of the closure load has been universally accepted. The most popular definition is the point where the load-displacement

trace becomes linear upon loading. Schijve[16] has argued that the corresponding crack closing load  $K_{cl}$  is more reproducible and less ambiguous. In practice, the difference ( $K_{op} - K_{cl}$ ) is often less than the scatter associated with closure measurements. A third convention is to locate the point of intersection of tangents drawn on the opening branch of the load versus closure transducer curve, Figure 4.5. One tangent corresponds to the fully open crack, the other to the fully closed crack. Whilst such a definition of the crack opening point is easily derived from the load-transducer trace, it is questionable whether the resulting point has any physical significance. At best, it may be argued that the tangency method yields a self-consistent set of closure data.

The change in specimen compliance associated with crack closure is often small. Kikukawa, Jono and Tanaka[17] have developed an offset procedure in order to aid discrimination of the crack opening load. The theory is straightforward, as follows.

For an open crack, the applied load  $P$  is proportional to any specimen displacement  $v$  (such as the crack mouth opening displacement), and we may write  $v = \alpha P$  where  $\alpha$  is the specimen compliance. We define the offset displacement  $\xi$  by  $\xi = k(v - \alpha P)$  where  $k$  is a gain of for example 20. Provided the crack is open  $\xi$  is zero and a plot of  $P$  versus  $\xi$  is a vertical line, see Figure 4.4. When the crack closes, the  $P$ - $v$  trace becomes non-linear and  $\xi$  becomes non-zero. The use of a high gain setting  $k$  enables the closure point to be detected on the load-offset displacement plot with high sensitivity and the minimum of subjectivity.

The offset procedure is realised in the crack closure measurement system by subtracting a fraction of the load signal from the displacement transducer signal using a simple summing amplifier, as shown in Figure 4.7. The resulting signal may be amplified by a factor of up to 1000 using the X-input amplifier of a digital storage oscilloscope, see Figure 4.7.

#### 4.4 CASE STUDY: CLOSURE RESPONSE OF BS4360 50B STEEL

A variety of compliance gauges have been described suitable for monitoring the closure response of a fatigue specimen. The question arises, do all the gauges indicate the same closure response? We answer this question here, for the case of a low strength structural steel, BS4360 50B. This material has a yield stress of 352 MPa, an ultimate tensile strength of 519 MPa and is of composition 0.14% C, 1.27% Mn, 0.41% Si, 0.017% P, 0.004% S, 0.073% Al, remainder Fe. In this section we examine the closure response of a specimen which is sufficiently thin (thickness=3mm) for the given loading ( $\Delta K = 25 \text{ MPa}\sqrt{\text{m}}$ , load ratio

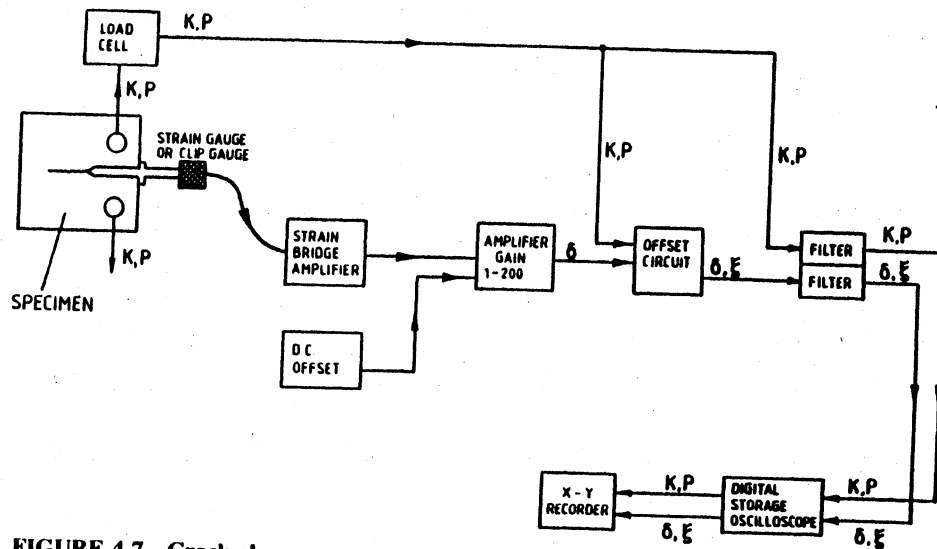


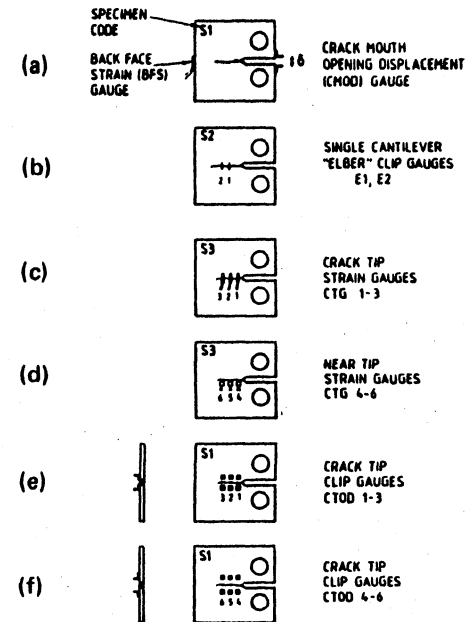
FIGURE 4.7 Crack closure measurement system.

$R=0.05$ ) that plane stress conditions prevail along the crack front. Then we consider closure under plane strain conditions, by examining the response of a much thicker test-piece (thickness=25mm) under the same  $K$  history. Finally, the progressive opening of fatigue cracks at loads below the closure load is measured for both types of specimen.

Tests were performed on 50mm wide compact tension (CT) specimens at a test frequency of 5 HZ. Loads were shed manually at crack growth increments of 0.25mm in order to maintain a constant stress intensity range and load ratio. The stress intensity range was of sufficient magnitude to give a crack growth rate  $da/dN \approx 10^{-4}$  mm/cycle. At these growth rates, crack closure is plasticity-induced for this material. Closure measurements were conducted every 0.5-1mm of crack growth over a range in crack length of 17.5-32mm. Load-displacement and load-offset displacement traces were recorded on a XY chart recorder, via a digital storage oscilloscope. Closure measurements were recorded at 0.02 Hz, and a 3rd order low-pass filter of cut-off frequency 1 Hz was used to reduce electrical noise. Full details on noise attenuation are given in Appendix A.

#### 4.4.1 Closure response of 3mm thick specimens (plane stress)

Three nominally identical CT specimens designated S1, S2 and S3 were tested under the same loading history. The location and gauge length of the displacement gauges and strain gauges used for taking compliance measurements are shown in Figure 4.8. Five types of gauge were used:



Gauge	Specimen code	Distance from loading axis (mm)	Gauge length (mm)
Back Face Strain Gauge (BFS)	S1, S2, S3	50	6
Crack Mouth Clip Gauge (CMOD)	S1	-14.7	5
Crack Tip Clip Gauge, CTOD 1	S1	16.25	5
Crack Tip Clip Gauge, CTOD 2		21.75	5
Crack Tip Clip Gauge, CTOD 3		27.25	5
Crack Tip Clip Gauge, CTOD 4	S1	16.25	9
Crack Tip Clip Gauge, CTOD 5		21.75	9
Crack Tip Clip Gauge, CTOD 6		27.25	9
Single Cantilever Elber Gauge, E1	S2	21.75	2
Single Cantilever Elber Gauge, E2		27.25	2
Crack Tip Strain Gauge, CTG 1	S3	16.25	5
Crack Tip Strain Gauge, CTG 2		21.75	5
Crack Tip Strain Gauge, CTG 3		27.25	5
Near Tip Strain Gauge, CTG 4	S3	16.25	1
Near Tip Strain Gauge, CTG 5		21.75	1
Near Tip Strain Gauge, CTG 6		27.25	1

FIGURE 4.8 Location of compliance gauges for 3mm thick compact tension specimens.

- (i) Crack mouth displacement gauge, labelled CMOD gauge.
- (ii) Back face strain gauge, labelled BFS gauge. The BFS gauge was used on the three specimens S1-S3 in order to detect any variation in closure response between the three specimens.
- (iii) Double cantilever crack tip displacement gauge, labelled CTOD1-6, and single cantilever crack tip displacement gauge, labelled E1 and E2. These gauges are positioned just behind the crack tip and straddle the fatigue crack. The twin cantilever gauge is mounted on the specimen with the aid of small anvils glued to the specimen before the commencement of the test. Each anvil is positioned to an accuracy of  $\pm 0.02$  mm with the aid of a jig. The single cantilever gauge is held against the side face of the specimen using rubber bands, and contacts the specimen by two needle points.

- (iv) Crack tip strain gauge, labelled CTG 1-3. These three gauges straddle the fatigue crack, just behind the crack tip. They are located in the same positions as the crack tip displacement gauges CTOD 1-3. Use of this type of gauge has been described in section 4.2.2.
- (v) Near tip strain gauge, labelled CFT 4-6. These gauges do not straddle the crack, but lie below the lower crack flank. Use of this type of gauge is given in section 4.2.2.

By positioning the gauges at a number of locations on the specimen it is possible to separate the influence of distance from gauge tip  $s$ , from the influence of crack length  $a$  upon the measured closure value  $U$ .

### Results

The back face strain gauges indicate similar closure responses for the three nominally identical specimens. We conclude that the three test-pieces may be considered identical for present purposes.

All displacement gauges and strain gauges with the exception of the near tip strain gauge give a  $U$  value equal to  $0.72 \pm 0.05$ , independent of distance of gauge from the crack tip and of the crack length, see Figure 4.9. The near tip strain gauges CTG 4-6 give  $U=0.72$  for  $s$  less than 2.75mm, see Figure 4.10. For greater values of  $s$ ,  $U$  increases gradually to unity. This is due to the fact that the near tip strain gauge does not bridge

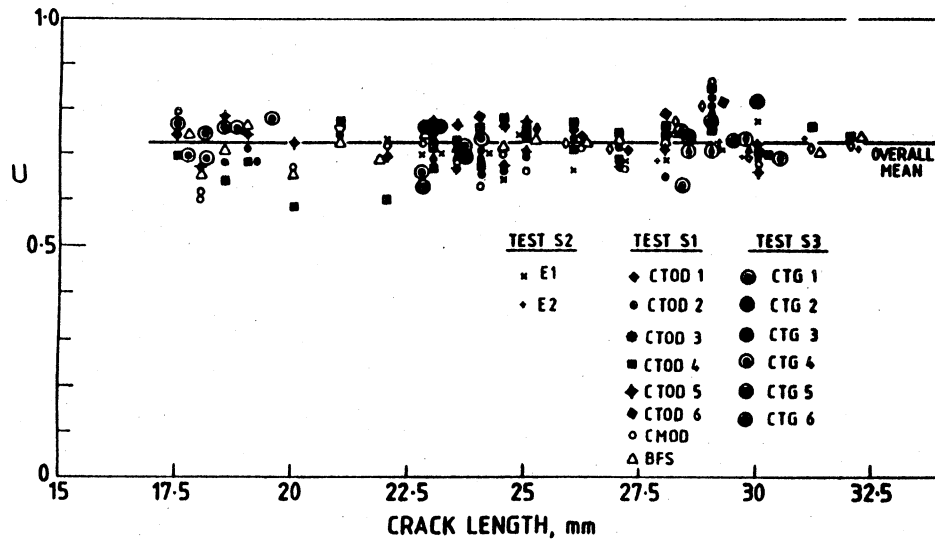


FIGURE 4.9 Effect of crack length upon fraction of load cycle for which crack is open,  $U$ , for 3mm thick specimens. For all gauges, observed closure response is independent of distance from gauge to crack tip.

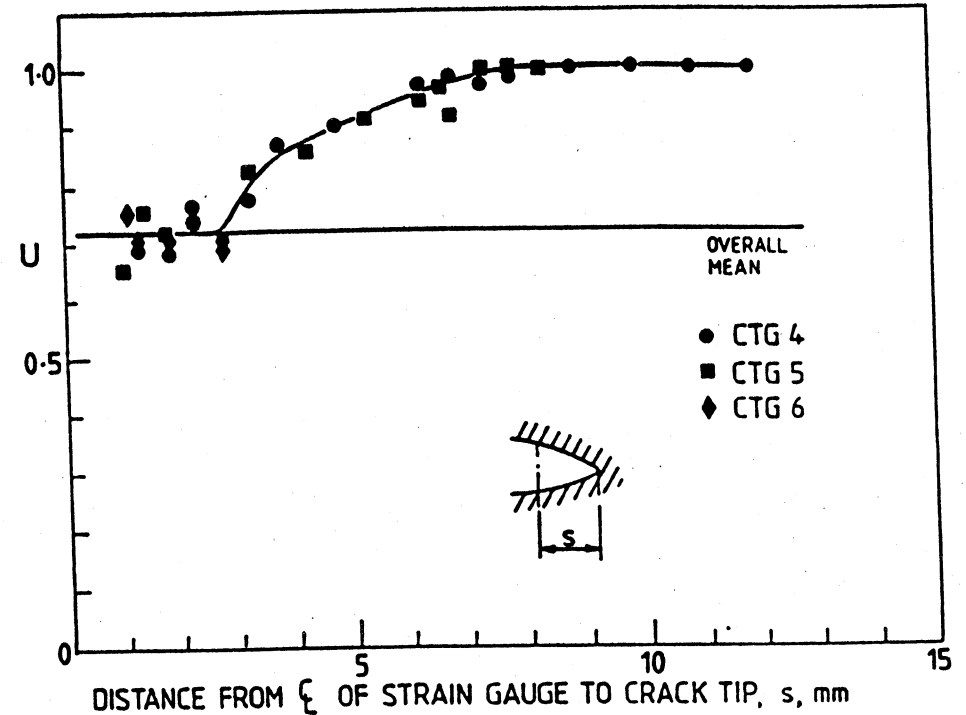


FIGURE 4.10 Variation of observed closure response with distance  $s$  from strain gauges CTG4-6 to crack tip. 24mm thick compact tension specimens.

the fatigue crack. The fatigue crack opens progressively to its tip and so we expect the  $U$  value to increase from 0.72 to unity with increasing distance  $s$  of gauge from the crack tip. We conclude that the near tip strain gauge is accurate provided the gauge is laid down at frequent intervals of crack advance: the method is prohibitively time consuming.

We conclude that the fraction of the load cycle for which the crack is open  $U$  is independent of compliance technique and crack length in these tests. The back face strain gauge is preferred, as it is simple to apply, shows little hysteresis and may be used to measure crack length in addition to the crack closure response.

#### 4.4.2 Closure behaviour of a 24mm thick specimen (plane strain)

The crack closure response has also been measured for two 24mm thick CT specimens, designated F1 and F2. These specimens are sufficiently thick for plane strain conditions to prevail along most of the crack front. The closure behaviour of specimen F1 was examined using a



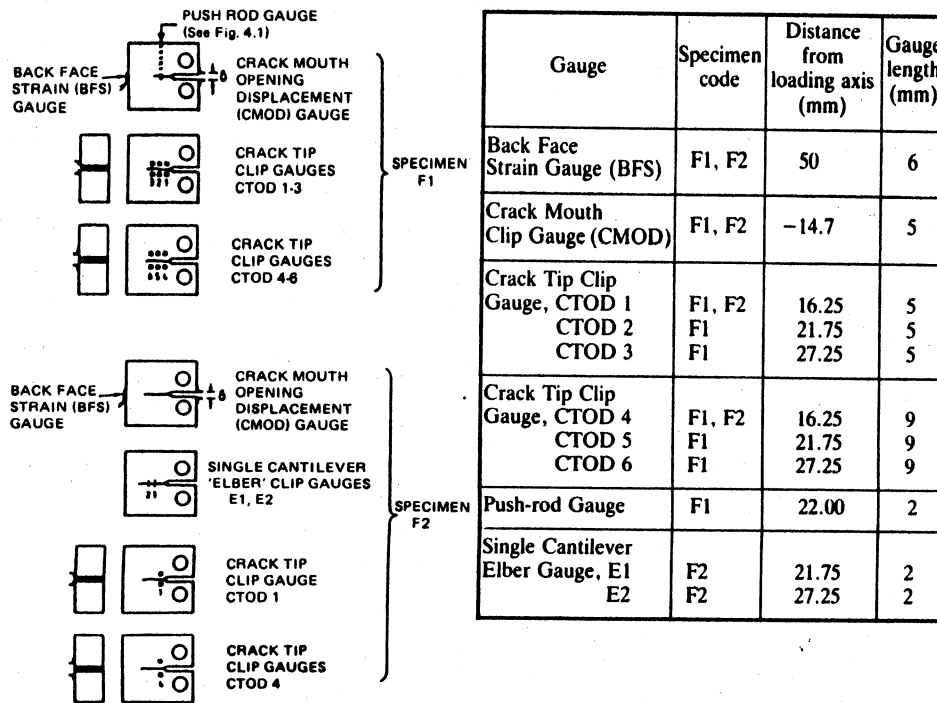


FIGURE 4.11 Location of compliance gauges for 24mm thick compact tension specimens.

crack mouth gauge (CMOD), back face strain gauge (BFS), double cantilever crack tip displacement gauge (CTOD 1-6) and a push rod gauge. Location of the gauges is given in Figure 4.11. Closure readings were taken once the crack grew out of the influence of the starter notch and until the crack attained a length of 23 mm. The test was then interrupted and the push rod gauge was mounted. After restarting the test, closure measurements were made using all gauges until the crack length attained 30mm when the test was stopped.

Closure readings were similarly taken for specimen F2 using a single cantilever displacement gauge (E1 and E2), double cantilever displacement gauges (CTOD 1,4), a crack mouth gauge (CMOD) and a back face strain gauge, see Figure 4.11. Gauges CTOD1 and 4, CMOD and BFS were used to check that there was no variation in closure behaviour for the test-pieces.

### Results

The fraction of the load cycle for which the crack is open  $U$  was

calculated from the load-offset displacement trace for each gauge. The two specimens F1 and F2 display identical behaviour to within experimental scatter. The push rod gauge, crack mouth gauge and back face strain gauge indicate a bulk, plane strain  $U$  value of 0.81, independent of distance from gauge to crack tip and independent of crack length. The back face strain gauge shows little hysteresis and is the preferred method for measuring the bulk crack closure response.

The measured closure value  $U$  for the near tip gauges is a function of distance from gauge to the crack tip. Provided the distance from the near tip gauge to the crack tip,  $s$ , is less than 3mm, the gauge indicates a surface, plane stress closure value  $U=0.71$ . (The plane stress plastic zone size  $r_p$  is approximately 1.7mm for the load level considered.) The  $U$  value increases to the bulk plane strain value of 0.81 as  $s$  is increased. This is illustrated in Figure 4.12 for the crack tip gauge CTOD 1-3.

We conclude that the  $U$  value for the surface, plane stress region of the 24mm thick specimen is  $U=0.71$  while  $U=0.81$  in the central, plane strain region of the specimen. The whole of the crack front in the 3mm thick specimens suffer plane stress conditions and  $U=0.72$ , independent of gauge type and gauge location. It appears that the surface closure response of the 24mm thick specimen is identical to the closure response along the whole crack front of the 3mm thick specimen. Results are summarised in Figure 4.13.

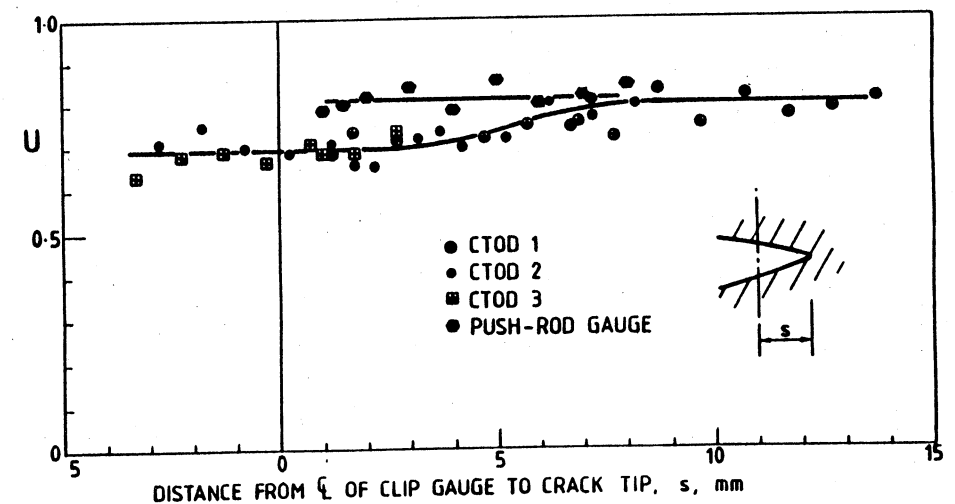


FIGURE 4.12 Effect of distance  $s$  from the push rod gauge and crack tip clip gauges CTOD1-3 to the crack tip upon the observed closure response. 24mm thick compact tension specimens.

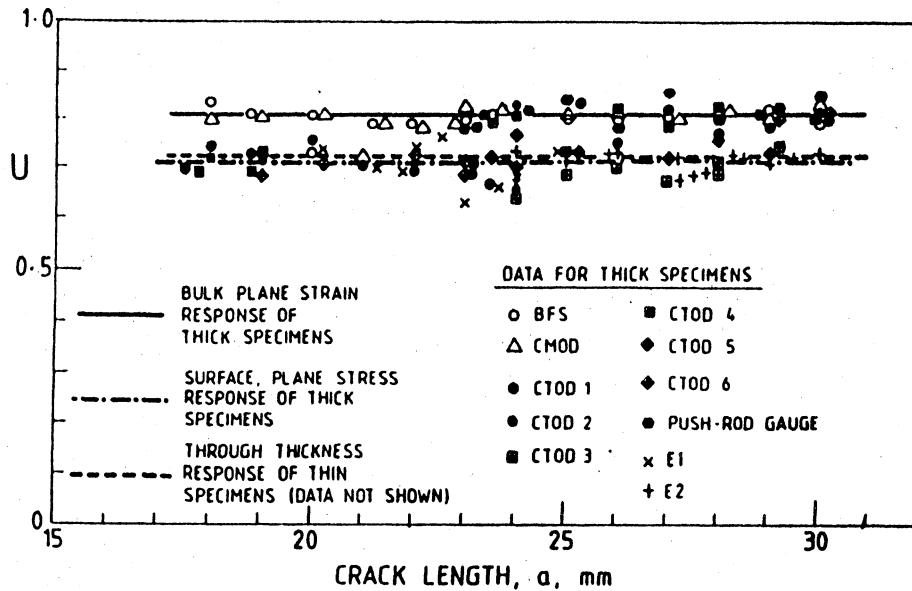


FIGURE 4.13 Influence of crack length on closure response for 3mm thick and 24mm thick compact tension specimens.

We expect the difference in closure responses between the two thicknesses of specimen to result in different crack growth rates. Is this the case? The average crack growth rate in the 3mm thick specimens is  $7.63 \times 10^{-5}$ mm/cycle, while the average growth rate for the 24mm thick specimens is  $1.80 \times 10^{-4}$ mm/cycle. If crack closure fully accounts for any difference in growth rate  $da/dN$  then in the mid-regime of the  $da/dN - \Delta K$  plot we expect the crack growth rate to satisfy the equation,

$$\frac{da}{dN} = C \Delta K_{\text{eff}}^n = C(U \Delta K)^n \quad (6)$$

where  $C$  is a constant, and a series of separate experiments show  $n=3.24$  for the BS4360 50B steel. Substitution of the measured bulk  $U$  values for the two specimen thicknesses into the above equation gives a predicted ratio of the growth rate in the 24mm thick specimen to the growth rate in the 3mm thick specimen of 1.46. Since the ratio of the measured growth rates is 1.42 we conclude that crack closure fully accounts for the faster growth in the thicker specimen.

#### 4.4.3 Measurement of progressive crack opening

The change in specimen compliance after a crack closes depends upon the extent of closure, which is measured by the closed crack increment.

When the closed crack increment is small then sensitive near tip compliance gauges are required in order to detect crack closure. The manner in which a crack opens and closes sheds further light on the mechanism and mechanics of crack closure. We can deduce the effective crack length, that is the length of open crack at any load, in the following ways:

- (1) *Compliance measurements.* The slope of the load-displacement, or load-offset displacement trace at any load corresponds to an open crack length. The open crack length is deduced from the stiffness versus crack length calibration for the particular geometry. Hence, we can infer the crack unpeeling response at loads below the crack tip opening load from back face strain and crack mouth compliance measurements. This has been done for specimen S2, see Figure 4.14. The loading and unloading portions of the load-displacement curves are very similar in shape for loads less than the crack tip opening load; thus, we argue that the crack opening and crack closing responses are almost identical.
- (2) *Direct observation of crack opening at various loads over a fatigue cycle,* using replication tape. This has been done for specimen S2 when the crack length attained a length of 24mm using the plastic replication technique outlined by Brown[18]. Full details have been given in Fleck[6]. From examination of the replicas in the scanning electron microscope, the crack opening profile is measured over a

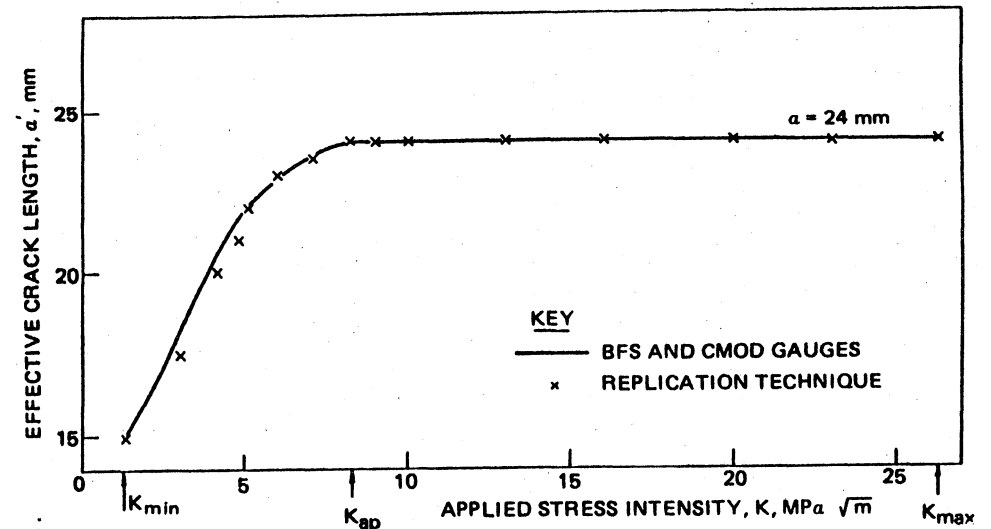


FIGURE 4.14 Comparison of crack flank opening response deduced from compliance and plastic replica methods.

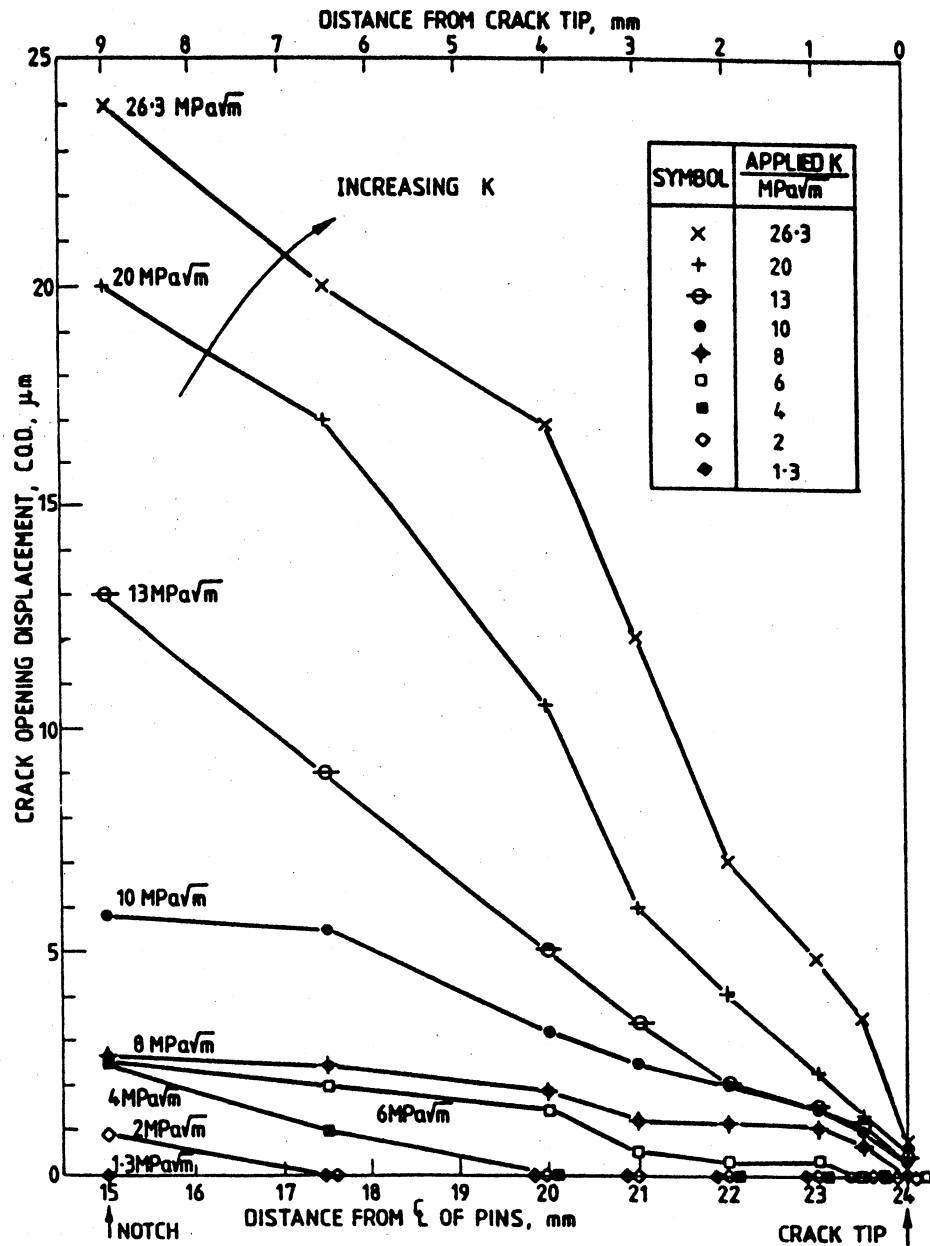


FIGURE 4.15 Crack opening response for 3mm thick specimen, by the plastic replica method.

fatigue loading cycle. The observed profile is given in Figure 4.15, and the associated crack opening response is included in Figure 4.14. We conclude from Figure 4.14 that the back face strain and crack mouth displacement compliance methods indicate the same effective crack lengths and the same crack tip opening load as the more direct replication procedure. This gives us confidence in the compliance technique.

We observed in section 4.4.2 that the back face strain (BFS) and crack mouth displacement (CMOD) methods give the crack closure response in the bulk, plane strain regions of a thick test-piece. We may therefore deduce the crack flank opening behaviour from the BFS and CMOD compliance traces. Results for specimen F1 are given in Figure 4.16, where the response for the 3mm specimen S2 are included for comparison. It appears that the closed crack increment is strongly dependent upon stress state. Under plane strain conditions, the crack is closed over approximately 1mm at minimum load of the fatigue cycle. Under plane stress conditions, the crack closes back to the notch length of 15mm at minimum load. Since the closed crack increment is small for the 24mm thick specimen, sensitive compliance instrumentation using the offset procedure and electrical filtering is required in order to measure the closure response.

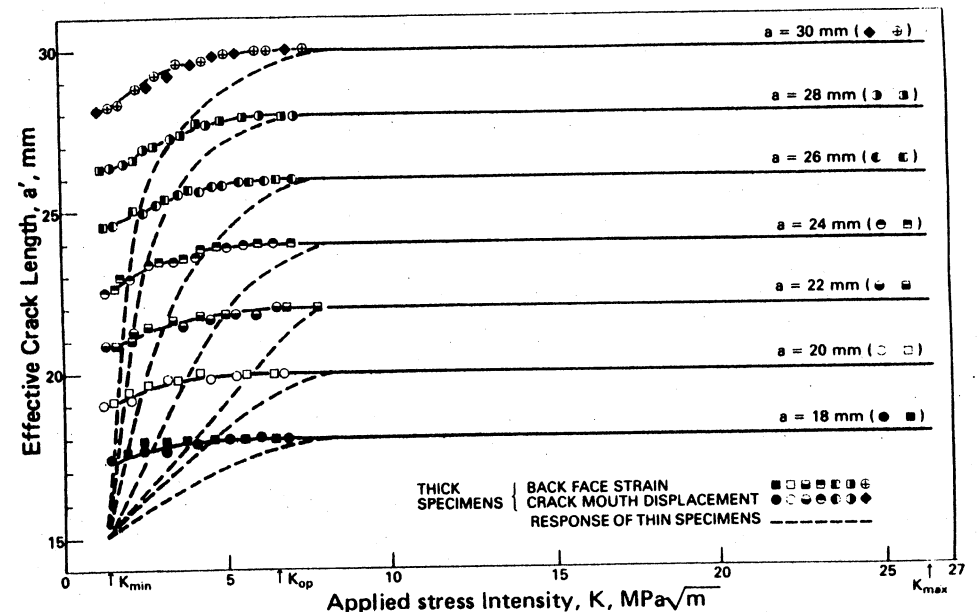


FIGURE 4.16 Crack flank opening response of 3mm thick and 24mm thick specimens, by compliance technique.

## 4.5 CONCLUDING DISCUSSION

The main conclusion from this chapter is that care must be exercised in the use of compliance methods to measure crack length and to monitor crack closure. The compliance gauge should be placed sufficiently close to a crack in order to have sufficient sensitivity. The change in compliance associated with crack growth and with crack closure is often small, and electrical filtering of the signal is required. An offset procedure is most helpful in aiding determination of the crack closure response. Compliance methods are a powerful means of elucidating the detailed manner in which a crack unpeels at loads below the crack opening load; measurements are in agreement with independent observations using a more laborious replication technique.

### Acknowledgement

The author is grateful for several helpful discussions with Prof. R.A. Smith.

### Appendix A. Filter network for compliance measurements

Usually the change in compliance due to the presence of a crack in a structure or specimen is small. This implies that the accuracy with which crack length or the crack closure load can be measured by compliance methods is highly dependent upon the noise levels superimposed on the load and displacement transducer signals. A low-pass filter may be used to attenuate noise to an acceptable level. The cut-off frequency of the filter should be between the signal frequency and the lowest frequency of unacceptable noise. Low frequency noise is often in the form of mains supply 50-60 Hz hum, and can be minimised by avoiding earth loops and by the use of screened leads.

The displacement transducer signal contains a number of harmonics, giving the load-displacement trace its non-linear shape. The low-pass filters must have a cut-off frequency greater than the highest harmonic of the displacement transducer signal. This is illustrated by the frequency spectrum of the signal from a back face strain gauge on a compact tension specimen, see Figure 4.A1. The specimen is 3mm thick, 50 mm wide and is made from BS4360 50B structural steel. Further test details are given in the figure. An analogue audio frequency analyser was used for the spectral density measurements. Signal power has been normalised with respect to total signal power at the test frequency of 5 Hz. We conclude from the figure that closure measurements must be conducted at sufficiently low frequency such that the first 5 harmonics of the

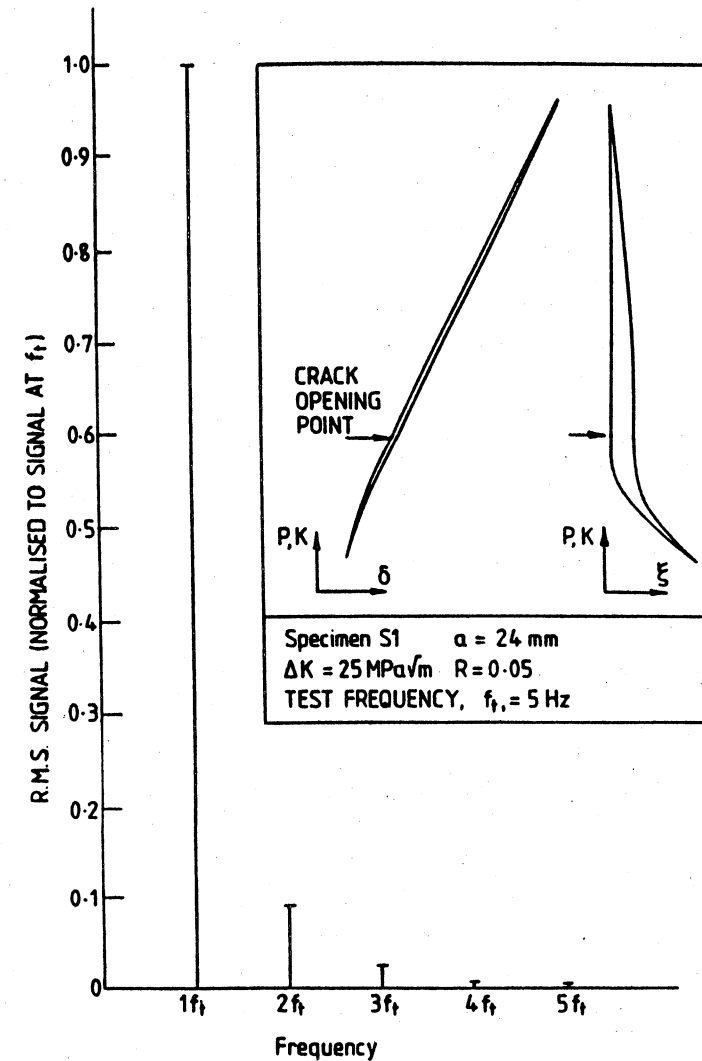


FIGURE 4.A1 Frequency spectrum of back face strain signal. 3mm thick specimen.

transducer signal are not affected by the low-pass filter. The maximum closure measurement frequency is dependent upon the particular low-pass filter employed. A suitable choice is a 3rd order passive low-pass filter with a -3db point at  $f_o = 1 \text{ Hz}$  as shown in Figure 4.A2. The filter shows phase distortion at a test frequency  $f \geq 0.1f_o$  and a loss in signal gain at  $f \geq 0.3f_o$ . Thus closure measurements should be taken at a frequency  $f \leq 0.1f_o$ .

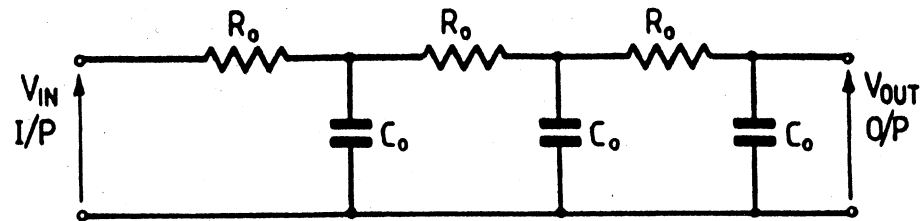


FIGURE 4.A2 Third order low-pass filter used in closure measurement system.  
 $R_o = 12k\Omega$ ,  $C_o = 2\mu F$ .

It is well known from filter theory (see for example [19]) that the higher the order of the filter, the greater is the phase distortion at frequencies below the cut-off frequency  $f_o$ . Thus, there is little advantage in employing a higher order active filter than the 3rd order passive Butterworths filter shown in Figure 4.A2.

#### References

- 1 Tada H, Paris P C, and Irwin G R. 'The stress analysis of cracks handbook', 2nd. edition, Paris Publications Incorporated, 1985.
- 2 Fleck N A. 'Some aspects of clip gauge design', Strain, February 1983, pp.17-22
- 3 Sullivan A M and Crooker T W. 'A crack opening displacement technique for crack length measurement in fatigue crack growth rate testing-development and evaluation', Engng Fracture Mech., Vol.9, 1977, pp.159-166.
- 4 Richards C E and Deans W F. 'The measurement of crack length and load using strain gauges', In The measurement of crack length and shape during fracture and fatigue, EMAS,1980, pp.28-68.
- 5 Fleck N A and Smith R A. 'Crack closure-is it just a surface phenomenon?', Int. J. Fatigue, July 1982, pp.157-160. Correction in Int. J. Fatigue, Oct.1982, p.243.
- 6 Fleck N A. 'An investigation of fatigue crack closure', Cambridge University Engineering Department report CUED/C-MATS/TR.104, May 1984.
- 7 Unangst K D, Shih T T and Wei R P. 'Crack closure in 2219-T851 aluminum alloy', Engng Fracture Mech., vol.9, 1977, pp.725-734.
- 8 Katcher M and Kaplan M. 'Effects of R factor and crack closure on fatigue crack growth for aluminium and titanium alloys', ASTM STP 559, 1974, pp.264-282.
- 9 Frandsen J D, Inman R V and Buck O. 'A comparison of acoustic and strain gauge techniques for crack closure', Int. J. Fatigue, vol.11, 1975, pp.345-348.

- 10 Mahulikar D S, Slagle W P and Marcus H L. 'Edge effects on fatigue crack closure of aluminium alloys', Scripta Met., vol.13, 1979, pp.867-870.
- 11 Mahulikar D S and Marcus H L. 'Fatigue crack closure and residual displacement measurements on aluminium alloys, Fatigue of Engng Mats and Structures, vol.3, 1981, pp.257-264.
- 12 Sewell G and Marcus H L. 'A model for fatigue crack closure based on surface roughness and residual strain', Scripta Met., vol.11, 1977, pp.521-524.
- 13 Bachmann V and Munz D. 'Crack closure in fatigue of a titanium alloy', Int.J.Fracture, Vol.11, 1975, pp.713-716.
- 14 Bachmann V and Munz D. 'Crack closure in fatigue crack propagation', Proceedings of Conf. on Fatigue Testing and Design, The Society of Environmental Engineering, Vol.2, 1976, pp.35.1-35.32.
- 15 Fleck N A. 'The use of compliance and electrical resistance techniques to characterise fatigue crack closure', Cambridge University Engineering Department report CUED/C-MATS/TR.89, January 1982.
- 16 Schijve J. 'Four lectures on fatigue crack growth', Engng. Fracture Mech., vol.11, 1979, pp.167-221.
- 17 Kikukawa M Jono, M and Tanaka K. 'Fatigue crack closure behaviour at low stress intensity level', Proc. ICM2, Boston, 1976, pp.254-277.
- 18 Brown C W. 'The initiation and growth of fatigue cracks in titanium alloys, Ph.D. Thesis, Cambridge University Department of Materials Science and Metallurgy.
- 19 Kuo F F. 'Network Analysis and Synthesis', 2nd edition, J. Wiley,1966.