K Control for Fatigue Crack Growth

ABSTRACT: This chapter describes an experimental technique which enables fatigue crack growth rate tests to be conducted at chosen values of the stress intensity factor, \( K \), independently of the crack length. The system uses the measured load combined with the back face strain (BFS) on a compact tension specimen (CT).

INTRODUCTION

In recent years the emphasis in experimental measurements of fatigue has swung from testing endurance as a function of applied stress to studying crack growth rates. This is a result of the growing application of fracture mechanics, in which the elastic stress field in the vicinity of a sharp crack is described by a single parameter \( K \), the stress intensity factor. Within certain well defined limits, the behaviour of the crack is determined by the behaviour of \( K \). In particular, fatigue crack growth rates for a given

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material in a given environment depend only on the alternating stress intensity factor \( \Delta K \), the difference between the maximum and minimum values of \( K \) in cyclic loading, and also to a lesser extent on the stress ratio \( R \), the ratio of the minimum to the maximum stress.

It follows that if a crack in a test specimen is subjected to a cyclic stress intensity factor whose maximum and minimum values remain constant, then the crack will grow at a constant rate. In many situations this is the most desirable way to measure crack growth rates. However, because of the difficulty of controlling \( K \) as an independent variable, crack growth tests have usually been measured at constant load amplitudes. For most specimen geometries \( K \) increases with crack length, and so the growth rate, \( da/dN \) increases as the test proceeds. Experimental records are made of crack length against loading cycles \( N \), and these are then differentiated to obtain \( da/dN \). The \( da/dN \) data is then plotted as a function of \( \Delta K \) on a log-log plot, from which useful power laws can be obtained. The great contribution of fracture mechanics is to enable the behaviour of cracks in engineering structures to be predicted from such data.

The scatter in tests carried out at constant load is, unfortunately, considerable, so that significant variations in growth rate at a given \( K \) can often be masked by the errors in crack length determination or in subsequent differentiation. Errors as great as a factor of between 2 and 10 are common (e.g. the data of Frost, Pook and Denton (1)). Such errors are often obscured by the ability of logarithmic plots to hide such variations by compressing the scales. This means that the effects on growth rate of secondary parameters, such as stress ratio, variable amplitude loading, environment and microstructure are often difficult to quantify with certainty. If tests can be carried out at controlled \( K \), then the growth rate will remain constant and can be measured with a much greater accuracy than in a test which is simply load controlled.

TECHNIQUES FOR CONSTANT \( \Delta K \)

Various attempts have been made to achieve constant \( \Delta K \), without changing the load cycle, by special geometry specimens. The first such test piece was the tapered double cantilever beam, invented by Mostovoy (2). However, it is expensive to manufacture and gives unstable crack growth direction (which is generally overcome by machining side grooves, but this leads to calibration
uncertainties). A more recent idea employed the fact that for a centre crack in a finite sheet, K increases with the crack length if a constant load is applied at the ends of the sheet, but decreases if the load is applied at points on the crack surfaces midway between the crack tips. By applying point loads through pins a small distance from the crack plane, a balance is found between these two effects (3). However, this type of specimen has the disadvantage that, for many materials, only modest values of K are obtained for the maximum allowable pin loads or before crushing of the specimen at the loading points begins.

If a servo-hydraulic rig is used for applying the fatigue load and if there is some means of monitoring the length of the crack, the load can be reduced as the crack grows in such a way as to keep $\Delta K$, and therefore the growth rate, constant. Monitoring may be done optically using a travelling microscope, in which case the load is shed manually (4), or using the electric P.D. method, in which case automatic control may be applied using a small computer (5). A standard geometry may be employed, the most generally useful being the ASTM Compact Tension Specimen, which is relatively inexpensive and permits high K values for relatively small machine loads. The disadvantage of manual load shedding is obvious: the electric potential method suffers considerable difficulties from stability and interference and, of course, can only be used for metals.

BACK FACE STRAIN GAUGE

In the first volume of 'The Measurement of Crack Length and Shape during Fracture and Fatigue' Richards and Deans described an elegant method of monitoring crack length in a CT specimen using a strain gauge cemented to the back face (6 - 8). At a given crack length this strain is proportional to K (hardly surprising, since K is defined for linear elastic systems). More importantly, for a given K this strain is also an approximately linear function of the crack length (Fig. 6 in their chapter). The method of K control described here follows directly from their work on the measurement of BFS.

If the load is known (this is a standard output on most servo-hydraulic machines) and also the BFS which is easily measured using a standard resistance strain gauge, then two relationships with two unknowns enable the crack length and K to be deduced, and thus load shedding can, in principle, be achieved automatically.
If microprocessor control is employed, this is most efficiently executed from the ASTM compliance relationship and the measurements of Richards and Deans.

ANALOGUE CONTROL

A happy relationship between the load and the BFS at a given value of K enables a very simple and inexpensive analogue control system to be used for CT specimens. This is illustrated in Fig. 1. The load required to produce a given K decreases with increasing crack length. On the other hand, the (compressive) BFS for a given K increases. It happens that both these lines are almost straight over the range of interest, but this is not in itself necessary for what follows. If these two relationships are summed in carefully chosen proportions, the result, i.e. $\alpha P + \varepsilon$ is a line approximately parallel to the abscissa. The additive constant, $\alpha$, must be right in order to maintain linearity over a wide range of crack lengths: too much of the load leads to a sum which decreases with crack length, too much of BFS the opposite. A change in proportion either side of the best value, $\alpha_0$, leads to a variation of the signal, and the variation that this would cause in the applied K using the system is shown in Fig. 2, for variations in $\alpha$ of $\pm 9\%$. It can be seen that there is some tolerance in the choice of this ratio, indeed small differences in the ratio merely serve to alter the range of optimum linearity.

A direct corollary of this result is that if the value of $\alpha P + \varepsilon$ is constant, then this corresponds to an approximately constant value of K, irrespective of the crack length. This is the basis of the analogue K control. A signal corresponding to $\alpha_0 P + \varepsilon$ will thus give a direct measure of K at the crack tip, accurate to $\pm 1\%$ over a range of $a/W$ of 0.3 - 0.6.

The principle of control in a servo-hydraulic test machine is the comparison of some feedback signal with the command signal by a servo amplifier. This operates the servo hydraulic valves so as to reduce the difference. If a K signal is used for the feedback in such a rig, then this will ensure that the stress intensity factor follows the command signal throughout the range of crack growth. For example, if the command signal is sinusoidal with constant amplitude, this should cause the crack to grow at constant K, and therefore at a constant growth rate. The system should also work for other closed loop control test rigs.
CUMULATIVE PLASTIC STRAIN

The major problem with this technique is that as fatigue crack growth proceeds there is a cumulative strain on the back face resulting from plastic deformations, this is shown in Fig. 19 of Richards' and Deans' Chapter in the previous volume. This is negligible in any single cycle, but over the large number of cycles in the practical tests it can grow to be significant compared with the cyclic elastic strain. The effect of this in the $K$ controlled system outlined above would be to cause the stress ratio to decrease. Therefore, although $\Delta K$ might remain constant, the mean value of $K$ would not, and in general this would affect the growth rate.

The cumulative strain problem is overcome by a separate circuit in the unit, designed to maintain a specified stress ratio, $R = \sigma_{\text{min}} / \sigma_{\text{max}}$. Two peak monitors indicate simultaneously voltages proportional to the maximum, $P_{\text{max}}$, and minimum, $P_{\text{min}}$, of the load signal. It is required to maintain $-R P_{\text{max}} + P_{\text{min}} = 0$. The $P_{\text{max}}$ signal is therefore amplified with a gain of $-R$ and added to the $P_{\text{min}}$ signal. The summed signal is then integrated and the final output is fed with appropriate sign to the $K$ control unit. This ensures full compensation for the cumulative plastic strain at the back face, and minimises any departure from the desired stress ratio.

The peak monitors are designed to respond immediately to changes away from a previous mean level, but to return to the desired level slowly. For reasons of stability (consider a fluctuation which caused a higher value of $P_{\text{max}}$) the stress ratio discrepancy signal must have a time constant which is very long compared with the peak monitor.

The constant $R$ facility may also be used to grow cracks with diminishing $\Delta K$, using the BFS signal only (i.e. with $\alpha = 0$). For this purpose a T-type WOL specimen may be preferred, since it gives a greater range of $\Delta K$ as the crack grows. This technique may be useful in threshold studies and for fatigue pre-cracking.

EXPERIMENTAL PERFORMANCE

$K$ control systems are now available commercially†; a $K$

† Details are available from ATECS, 339 Halesowen Road, Cradley Heath, Warley, West Midlands, U.K.
controller fitted into an Instron servohydraulic rig is shown in Fig. 3(a); this one is specially designed to be fully compatible with Instron's interlocks between modules, and also to allow conventional strain control to be used. Fig. 3(b) shows a close-up of the specimen, with a strain gauge cemented to the back face, together with a compensating dummy gauge (a half bridge has been found satisfactory for most applications). A scale is also attached for visual observation of the crack length (for the data in Fig. 4, a travelling microscope was used to give higher accuracy). The K controller itself is shown in Fig. 3(c). It consists of push buttons for selecting conventional strain control or K control, with indicating LED's, three 10-turn potentiometers for setting α (strain scaling), gain, and R (stress ratio), and two LED's and a three position switch for setting up and controlling the stress ratio. Setting up the system requires only a few minutes, and once it has been set up for a given specimen geometry and material no further adjustments are necessary for subsequent tests (except that R may be altered if required).

To prove itself the K controlled system must satisfy two separate requirements; first, the a against N plot for a given test must be straight for a substantial range of crack length, second, the growth rate under nominally identical conditions must be reproducible.

Typical measurements are shown in Fig. 4. Initially the crack growth is slow, partly because it is growing from a notch of nominal radius 0.25 mm, and also because the crack grows faster in the centre of the plate than at the surface, where optical measurements are made. (The BFS probably gives a better indication of mean crack length than the optical surface measurements). Towards the end of the life of the specimen the crack again slows down, here because the load is so small that strain dominates the feedback signal. However, between a/W = 0.32 and a/W = 0.6 the growth is tolerably linear. (W is the specimen width, from loading holes to back face). For the material tested it was found that da/dN ∝ ΔK^3, so that a departure from linearity of 3% in the growth rate corresponds to 1% in ΔK.

Once the method has been established, fewer measurements need to be made than are indicated by Fig. 4. This is an additional advantage over load controlled tests, where readings must be taken at frequent intervals, increasingly so as the test proceeds and the crack growth accelerates.

The test was repeated under nominally identical conditions.
including environment. The measurements were independently analysed, and the growth rate thus determined differed by less than 7% from that found previously. This would indicate less than 2\(1/2\)% error in \(K\) even if there were no other errors, but some do also arise from the optical measurements and subsequent analysis, and possible variation in the material itself.

CRACK CLOSURE LIMITATIONS

If crack closure occurs and increases in magnitude during the test, an increase in stress intensity factor will be experienced. This is because the strain signal, \(\varepsilon\), will be reduced, see Fig. 5, and the load will therefore increase to keep the sum \((aP + \varepsilon)\) constant. From a practical point of view the problems are slight, since the applied load range may be checked at the beginning and end of the test. For the material of Fig. 4, no closure was found for \(R > 0.1\).

To consider the significance of crack closure upon the test technique, suppose that no closure were present at the start of a test with \((a/W) = 0.3\), but by the end, \((a/W) = 0.6\), some closure occurred reducing the effective load range to a fraction, \(U\), of its total. Fig. 6 gives computed values for the increase in \(\Delta K\), which strongly depends on the distance behind the crack tip, \(\Delta a\), over which the crack is closed. If the crack closes right back to the machined notch tip (\(\Delta a = 15\) mm in our example) then for \(U = 0.9\), the error in \(\Delta K\) is 6\%, but for \(U = 0.7\), this will increase to about 20\%. However, recent work (9) suggests that \(\Delta a\) in the bulk, plane strain region of the crack tip is much reduced from its surface plane stress value. If \(\Delta a\) is, say, 3 mm, then the errors above reduce to 2 and 7\% respectively. Since these computations are based on grossly pessimistic extremes of behaviour, it may be concluded that the problems of closure are not severe, provided that sufficient care is taken with the interpretation of any non-linearity of the initial BFS record.

THE USES OF \(K\) CONTROL

The accuracy obtained with the \(K\) control system, about 7\% on growth rate, may be favourably compared with the scatterband described in the introduction, which was as much as a factor of 10 on rate. Therefore, where it is desirable to obtain detailed knowledge of fatigue crack growth behaviour there is much advantage to
be gained by K controlled testing. The system described offers accuracy and simplicity combined with stability and low costs. It is hoped that this will facilitate the experimental side of studies directed to a better understanding of the effects of the many parameters on which fatigue crack growth depends.

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REFERENCES


FIG. 1. The principle of the $K$ feedback signal. The falling load ($P$) and rising BFS ($\varepsilon$) signals are summed with appropriate constant, $a$, to give a signal which is proportional to $K$ and independent of the value of $a/W$. 

$\alpha P + \varepsilon$ (constant)

$\varepsilon$ (proportional to BFS)

$P$ (proportional to load)

Crack length $a/W$
FIG. 2. The value of $K$ for a constant signal $aP + \epsilon$, for different values of $a$. Optimum linearity is achieved with $a = a_0$ (central curve), the top and bottom curves correspond to changing $a$ by $\pm 1\%$. 
FIG. 3(a) X controller fitted to a servohydraulic test rig. Figs. 3(a), (b) and (c) are courtesy of Mr. R. Martin of Instron Limited.
FIG. 3(b) CT specimen with BFS gauge.
FIG. 3(c) K controller panel.
FIG. 4. Experimental measurements at nominally constant $\Delta K$ and $R$ using the $K$ controller. The crack length was measured optically, using a travelling microscope. Even closer fit to a straight line can be obtained by restricting observations to $a/W \leq 0.6$.

FIG. 5. The effect of closure on the load/strain relationship.
FIG. 6. Error due to extreme case of crack closure; zero closure at $a/W = 0.3$, crack open for a fraction $U$ of the applied load range at $a/W = 0.6$. 