UDC 53.084

Some aspects of clip gauge design

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Measurements show that simple beam theory accurately models the performance of clip gauge transducers. Assuming typical dimensions for the single and twin cantilever gauge it is found that the single cantilever gauge possesses about four times the sensitivity of the twin cantilever gauge, for the same stiffness. Design rules are specified by which gauges of high sensitivity and low stiffness may be constructed. A simple and reliable technique for fastening clip gauges to a test piece is introduced.

Key words: Strain gauge, clip gauge, displacement transducer

Introduction

The need to measure small displacements with great accuracy and reliability has encouraged the development of several types of displacement transducer. For many applications the clip gauge has distinct advantages over the linear voltage displacement transducer, capacitance gauge and interferometric methods. The clip gauge is small, light, robust and inexpensive. Unfortunately, its design appears to be highly heuristic in nature. The aim of this paper is to rationalise the design of clip gauges, and thus remove the mystique which surrounds the use of these devices.

Previous work

The need to measure crack opening displacements of less than 1 mm in fracture toughness tests spurred development of the twin cantilever clip gauge.[1] The design of the gauge was based on an optimisation of the following parameters: (1) the sensitivity, (2) the useful measurement range, (3) the difference in slopes of the ends of the two beams when attached to a specimen, and (4) the attaching force of the gauge. Two standard gauges evolved, an American design recommended by the American Society for Testing and Materials[2] and a British design recommended by the British Standards Institution.[3] The British gauge has short, thin and tapered arms in comparison with the American gauge, although it has a similar performance. These transducers give linear, repeatable outputs, and are robust and simple to use. Unfortunately, they are not suitable for applications where the measured displacements are minimal and the gauge attachment force is to be small. Such applications demand a gauge of high sensitivity and low stiffness, properties which are easily predicted using simple theory.

Theory

Two commonly used forms of clip gauge are considered, the single and twin cantilever gauges.

Single cantilever gauge

The stiffness and sensitivity of the single cantilever gauge can be determined from simple beam theory.[4] An idealised gauge is depicted in Fig. 1.

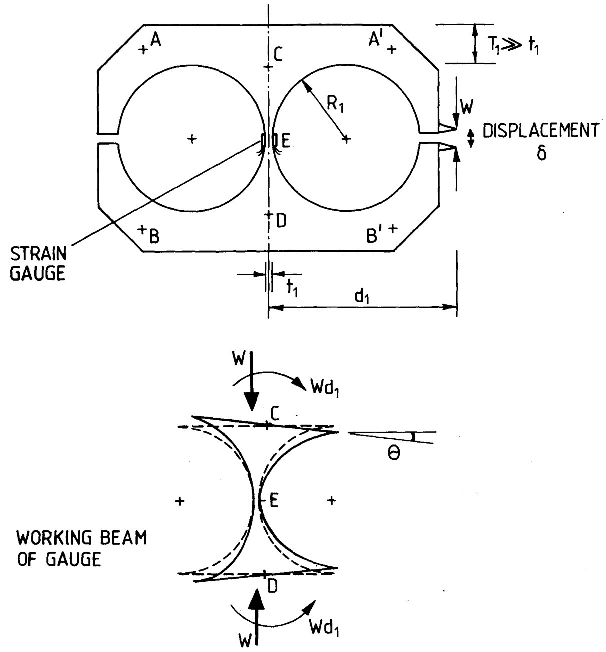

Fig. 1 Idealisation of single cantilever gauge

Consider first the stiffness of the gauge. The stiffness is defined as the force required to deflect the needle points of the gauge by unit displacement. It is assumed that the connecting arms *AA'* and *BB'* behave as rigid bodies (Fig. 1). On deflecting the needle points of the gauge by a displacement δ each end of the beam *CD* deflects by an angle θ, where

$$\theta \simeq \frac{\delta}{2d_1}. \quad (1)$$

The moment experienced by each end of the beam is Wd_1, where W is the applied load and d_1 is the distance from the needle point to beam.

In order to calculate the stiffness of the gauge it is necessary to determine the relation between the moment, Wd_1, applied to the beam *CD*, and the associated end slope, θ.

The second moment of area, I, at any section is given by

$$I = \frac{1}{12} b_1 (t_1 + 2\xi)^3 \quad (2)$$

where b_1 is the width of the beam and $(t_1 + 2\xi)$ is

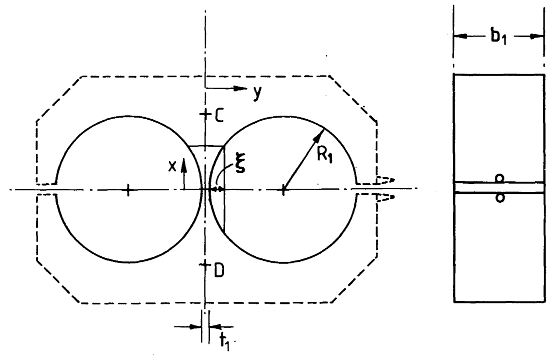

Fig. 2 Definition of co-ordinates for determination of stiffness of beam *CD*, single cantilever gauge

the depth (Fig. 2). The increase in depth, 2ξ, with increasing distance, x, from the centre section of the beam *CD* is deduced from the intersecting chords theorem,

$$\xi(2R_1-\xi)=x^2. \tag{3}$$

The change in curvature of the beam, d^2y/dx^2, is greatest near the mid-section, where $\xi \ll 2R_1$, and so equation (3) can be simplified to the form,

$$2R_1\xi \simeq x^2 \tag{4}$$

The moment-curvature relation for a beam in bending is given by,[4]

$$M=EI\frac{d^2y}{dx^2}. \tag{5}$$

Here, M denotes the applied moment and E is the Young's modulus.

The end slope, θ, may be deduced by integrating the curvature, d^2y/dx^2, with respect to x, from mid-section of the beam *CD* to location *C* (Fig. 2). Thus,

$$\theta=\int_0^{R_1}\frac{d^2y}{dx^2}\,dx. \tag{6}$$

Combining equations (2), and (4) to (6) and substituting Wd_1 for M, θ may be expressed as

$$\theta=\int_0^{R_1}\frac{12Wd_1}{E_1b_1}\cdot\frac{1}{\left(t_1+\frac{x^2}{R_1}\right)^3}\,dx. \tag{7}$$

Equation (7) may be integrated analytically using the substitution,

$$\tan\phi=\frac{x}{\sqrt{t_1R_1}}. \tag{8}$$

The variable ϕ takes the limits,

lower limit,

$$x=0\rightarrow\phi=0$$

upper limit,

$$x=R_1\rightarrow\phi=\tan^{-1}\left(\frac{R_1}{\sqrt{t_1R_1}}\right)\simeq\pi/2,$$

since $R_1 \gg t_1$.

Hence equation (7) reduces to,

$$\theta=\frac{12Wd_1}{E_1b_1t_1^3}\sqrt{t_1R_1}\int_0^{\pi/2}\cos^4\phi\,d\phi$$

$$=\frac{9\pi}{4}\frac{Wd_1\sqrt{t_1R_1}}{E_1b_1t_1^3}. \tag{9}$$

The stiffness of the single cantilever gauge, W/δ, is derived by eliminating θ from equation (1) and (9), yielding

$$\text{stiffness}=\frac{W}{\delta}=\frac{2}{9\pi}\cdot\frac{E_1b_1t_1^3}{d_1^2\sqrt{t_1R_1}}. \tag{10}$$

The sensitivity of the clip gauge is defined as the ratio of the strain, ε_E, experienced by the outer fibre of the beam at location E, to the end deflection, δ. By simple beam theory,[4] the strain ε_E is given by

$$\varepsilon_E=\frac{My}{EI} \tag{11}$$

where M is the bending moment experienced at the particular section, y is the distance from neutral axis to surface fibre of the beam, and EI is the bending stiffness. At the section under consideration, $M=Wd_1$, $y=t_1/2$ and $I=\frac{1}{12}b_1t_1^3$, hence

$$\varepsilon_E=\frac{(Wd_1)(t_1/2)}{E_1(\frac{1}{12}b_1t_1^3)}. \tag{12}$$

Upon elimination of W using equation (10), the sensitivity is given by

$$\text{sensitivity}=\frac{\varepsilon_E}{\delta}=\frac{4}{3\pi}\cdot\frac{t_1}{d_1\sqrt{t_1R_1}}. \tag{13}$$

Twin cantilever gauge

The sensitivity and stiffness of a twin cantilever beam gauge are determined by modelling one arm of the beam as shown in Fig. 3. It is assumed that the strain gauges suffer the same strain as that experienced at point A, located at a distance $l_2/4$ from the encastré end of the gauge. In addition, the curvature of the portion BC of the arms is assumed to be negligible in comparison with that of portion OB. This assumption is justified by the following arguments:

1. The bending stiffness EI of the portion BC is nearly an order of magnitude greater than that of portion OB.
2. The average bending moment experienced by BC is one third that experienced at point A.

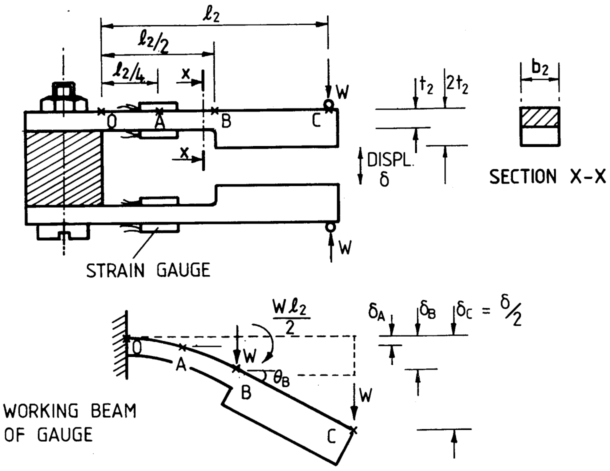

Fig. 3 Idealisation of twin cantilever gauge

The stiffness of the gauge can now be determined. It is known from simple beam theory[4] that the end deflection of a built-in beam of length L, stiffness EI, subject to a load W is given by $WL^3/3EI$, whilst the end slope is $WL^2/2EI$. Similarly, the end deflection of a built-in beam subjected to an end moment M, is given by $ML^2/2EI$; the end slope is given by ML/EI. Using these deflection coefficients, the end deflection δ_c, of the beam OC may be determined:

$$\delta_c = \delta_B + (l_2/2)\theta_B \tag{14}$$

where δ_B is the deflection of point B and θ_B is the slope at point B.

Now

$$\delta_B = \frac{W(l_2/2)^3}{3E_2I_2} + \frac{\left(\frac{Wl_2}{2}\right)\left(\frac{l_2}{2}\right)^2}{2E_2I_2}. \tag{15}$$

and

$$\theta_B = \frac{W(l_2/2)^2}{2E_2I_2} + \frac{\left(\frac{Wl_2}{2}\right)\left(\frac{l_2}{2}\right)}{E_2I_2}. \tag{16}$$

Hence by equations (14–16),

$$\delta_c = \frac{7}{24}\cdot\frac{Wl_2^3}{E_2I_2}. \tag{17}$$

The second moment of area, I_2, is given by $\frac{1}{12}b_2t_2^3$. It can be concluded that the stiffness of the twin cantilever gauge is

$$\text{stiffness} = \frac{W}{2\delta_c} = \frac{1}{7}\cdot\frac{E_2b_2t_2^3}{l_2^3}. \tag{18}$$

A relation for the sensitivity of the gauge can now be derived. The sensitivity is defined as the ratio of the strain ε_A experienced by the outer fibre of the beam at location A, to the end displacement, $2\delta_c$. The bending moment, M, at position A is given by $3Wl_2/4$; hence, by equation (11),

$$\varepsilon_A = \frac{(3Wl_2/4)(t_2/2)}{E_2I_2}. \tag{19}$$

Upon elimination of W using equations (18) and (19), the sensitivity of the gauge is expressed by,

$$\text{sensitivity} = \frac{\varepsilon_A}{2\delta_c} = \frac{9}{14}\cdot\frac{t_2}{l_2^2}. \tag{20}$$

The theoretical stiffness and sensitivity of the two types of gauges are summarised in Table 1.

Comparison of theoretical and measured performances of clip gauges

In order to judge the accuracy of the above theory, the stiffness and sensitivity of a single and twin cantilever gauge were measured; dimensions of the gauges are given in Table 2. The sensitivity of each gauge was determined by imposing a displacement on the clip gauge arms with a conventional barrel micrometer, and noting the corresponding strain gauge reading. The stiffness was deduced by loading the clip gauge arms with dead weights and monitoring the associated strains; displacements were inferred from the previous sensitivity calibration.

Fair agreement was found between the theoretical and measured performance of gauges, as evidenced in Table 3. It is concluded that simple theory adequately describes the response of both the single beam and twin beam clip gauge. It is, therefore, legitimate to compare the performance of the two types of gauge and construct a number of design rules, using simple theory.

Table 1 Theoretical performance of clip gauges

Gauge	Stiffness	Sensitivity	$\phi\left(=\frac{\text{sensitivity}}{\text{stiffness}}\right)$
Single cantilever	$\frac{2}{9\pi}\cdot\frac{E_1b_1t_1^3}{d_1^2\sqrt{t_1R_1}}$	$\frac{4}{3\pi}\cdot\frac{t_1}{d_1\sqrt{t_1R_1}}$	$\frac{6d_1}{E_1b_1t_1^2}$
Twin cantilever	$\frac{1}{7}\cdot\frac{E_2b_2t_2^3}{l_2^3}$	$\frac{9}{14}\cdot\frac{t_2}{l_2^2}$	$\frac{9}{2}\cdot\frac{l_2}{E_2b_2t_2^2}$

Table 2 Dimensions of single and twin cantilever clip gauges

	Single cantilever gauge				Twin cantilever gauge		
Symbol	R_1	d_1	t_1	width b_1	l_2	t_2	width b_2
Size (mm)	4·9	18	0·50	6·0	20	0·9	3·0

Gauges made from 7075-T6 aluminium alloy.
Young's modulus, E, =70 GPa. Yield stress, σ_y, =500 MPa.

Table 3 Comparison of measured and theoretical performances of clip gauges
Dimensions of gauges are given in Table 2

	Stiffness (N/mm)		Sensitivity ($\mu\varepsilon/\mu$m)	
Gauge	Theoretical	Measured	Theoretical	Measured
Single cantilever	7·32	6·83	7·53	6·78
Twin cantilever	2·73	2·93	1·45	1·13

Comparison of single and twin cantilever gauges

A clip gauge of high sensitivity and low stiffness is desired when measuring small displacements. It is, therefore, useful to determine which type of cantilever gauge possesses the greater sensitivity for a given stiffness.

In accordance with equations (10) and (18), the single and twin cantilever gauges have the same stiffness provided

$$\frac{2}{9\pi}\cdot\frac{E_1 b_1 t_1^3}{d_1^2\sqrt{t_1 R_1}}=\frac{1}{7}\cdot\frac{E_2 b_2 t_2^3}{l_2^3}. \qquad (21)$$

Ascribing typical values to the terms E_1, E_2, d_1, R_1, l_2 and t_2 of equation (21), a value for t_1 may be deduced such that the two clip gauges possess the same stiffness. For example,

$E_1=70$ GPa	$E_2=70$ GPa
$d_1=18$ mm	$l_2=20$ mm
$R_1=4{\cdot}9$ mm	$t_2=0{\cdot}9$ mm
$b_1=6{\cdot}0$ mm	$b_2=3{\cdot}0$ mm

From equation (21), the minimum thickness t_1 is 0·34 mm.

Finally, the dimensions of the two gauges are substituted into equation (13) and (21), yielding

$$\frac{\text{sensitivity of single cantilever gauge}}{\text{sensitivity of twin cantilever gauge}}=4{\cdot}3.$$

Thus, for a specified stiffness, the single cantilever gauge possesses more than four times the sensitivity of the twin cantilever gauge.

Rules for the design of clip gauges

It is apparent that sensitivity and stiffness are not independent functions of gauge geometry: inevitably, sensitive gauges are also stiff gauges. The ratio, ϕ (=sensitivity/stiffness) for each design of gauge is included in Table 1. In order to maximise the parameter, ϕ, for a gauge of high sensitivity and low stiffness, the requirements are as follows.

1. Design as small a gauge as possible. The term ϕ scales with the inverse square of the size of geometrically similar gauges.
2. Use a material of low Young's modulus, E, in construction. Aluminium alloys and titanium alloys are therefore more suitable than steels.
3. Ensure that the gauge is narrow (minimise b_1, b_2) with long arms (maximise R_1, l_2) and a thin working section (minimise t_1, t_2).

In addition, the following points should be observed for a gauge of high electrical sensitivity, large working range and low thermal drift/noise.

(i) Place the strain gauges as near to the point of maximum bending moment as possible. For the case of a single cantilever gauge, the bending moment is constant along the working beam; for the twin cantilever gauge, the maximum bending moment is experienced at the built-in ends of the two beams.

(ii) Concentrate beam curvature in the region of the strain gauges. Thus, the connecting arms *AA′* and *BB′* of the single cantilever gauge (Fig. 1) are made much stiffer than the working section. Similarly the portion *BC* of the arms of the twin cantilever gauge (Fig. 3) is made much stiffer than the working portion *OB*. It is easily shown that a twin cantilever gauge with arms of uniform thickness, *t*, has only 0·75 times the sensitivity of the equivalent twin cantilever beam depicted in Fig. 3. The British Standard for fracture toughness testing[3] recommends the use of clip gauge beams which increase in thickness from specimen end to encastré end. Whilst such a design ensures minimal risk of overstraining the gauge, the sensitivity is reduced.

(iii) Use a material of high yield strain, e.g. 7075-T6 aluminium alloy or Ti–6Al–4V titanium alloy. This maximises working range by minimising the risk of overstraining the gauge.

(iv) Employ a fully active strain gauge bridge, made from temperature compensated strain gauges; also, thermally insulate the gauge using e.g. cotton wool. Short and long term temperature drifts are thus minimised. The use of 350 Ω or 500 Ω strain gauges allows for high bridge voltages (up to 10 volts) and maximum electrical sensitivity of the strain gauges.[5]

(v) Screen the wires between strain gauges and strain bridge amplifier to minimise electrical noise.

To be successful, a clip gauge must also be designed to endure its operational conditions, such as environment and temperature. The device should be matched to the type of strain gauge and associated instrumentation. Care must be taken in the use of bolted joints, and in the detail design of the mounting method.

Mounting of clip gauges

The method by which a clip gauge is fastened to a test piece is not limited by the type of gauge. Rather, the fixing technique is dependent upon whether one wishes to measure the displacement between two points, or two larger areas, on a test piece. When the displacement between the two points is required, the gauge may be located using needle points, and held to the specimen by tension springs or rubber bands (Fig. 4a). Location of the needle points may be aided by indenting the specimen with a Vickers hardness indenter.

The average displacement between two areas is measured by bonding a pair of knife edges or anvils to the test piece. The knife edges fit into a machined groove on each arm of the clip gauge, and thereby fasten the transducer to the specimen (Fig. 4b). Alternatively, the gauge locates on to the anvils by universal joints, comprising a hole in the anvil and a mating steel ball bonded to each arm of the gauge (Fig. 4c).

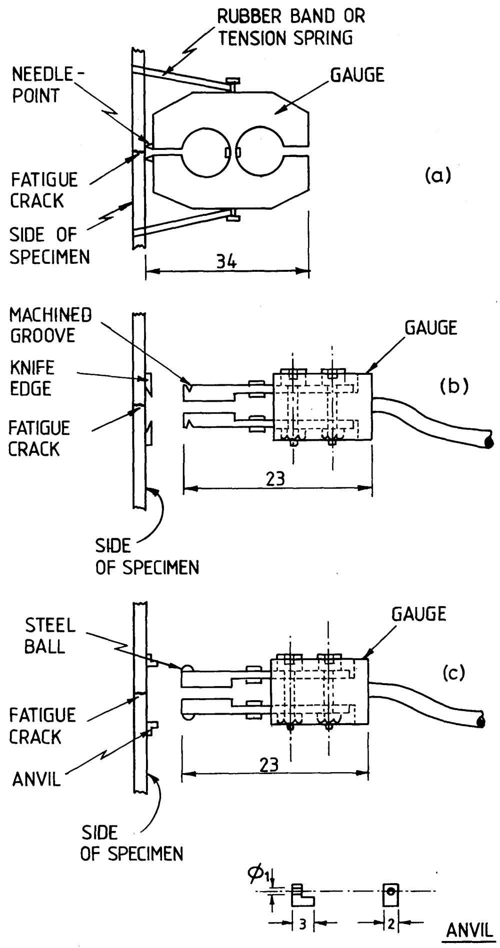

Fig. 4 Mounting techniques for clip gauges: (a) Needle points; (b) knife edges; and (c) ball and socket joints. All dimensions in mm

Provided that measurement of an averaged displacement is acceptable, the use of knife edges or anvils is recommended as these mounting methods are much more positive and reliable than the use of needle points. The current standards on fracture toughness testing[2,3] advocate the use of knife edges to minimise rotational friction effects and associated nonlinearity of the load-displacement record. No evidence of rotational friction was found using the ball and socket coupling. It is concluded that the ball and socket arrangement is best, since this does not suffer from misalignment problems.

Conclusions

1. Simple beam theory may be used to predict the performance of single and twin cantilever clip gauges.
2. Typically, for a given stiffness, the single cantilever gauge is more sensitive than the twin cantilever gauge.
3. A set of design rules for clip gauges may be constructed from theoretical considerations. Such design rules simplify and rationalise gauge development.
4. Careful consideration should be given to the method by which displacement transducers are

fastened on to a test-piece. A novel mounting technique, which employs ball and socket joints, provides for positive, reliable and noise free measurement of displacement.

Acknowledgements

The author wishes to thank Dr R. A. Smith and Mr M. C. Smith for helpful discussions, the head of Cambridge University Engineering Department for provision of test facilities, and the Department of Education, Northern Ireland for financial support.

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