Compressive kinking of fiber composites: A topical review

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This is a review of the results of several selected theoretical studies concerned with the localized kinking, or microbuckling, of aligned fiber composite materials subjected to compression in the fiber direction. Compressive kinking is of primary concern in polymer matrix composites, for which kinking failure can limit the compressive strength to a value that is usually much lower than the tensile strength. A similar situation can occur in carbon matrix fiber composites.

Compressive kinking failure may be understood on the basis of an elementary theoretical approach that ignores the influence of bending resistance of the reinforcing fibers, but takes into account the nonlinearity of composite constitutive relations as well as the effects of initial imperfections in fiber alignment. Kink bands bounded by fiber breaks are produced by deformations that occur after the attainment of peak compressive loads. The theoretical calculation of the widths of such kink bands does require consideration of fiber bending resistance; on the other hand, the results for kink width are not sensitive to the sizes of initial fiber misalignments. Progress in the study of the following additional kinking topics is summarized briefly: correlation of static kinking strength and random fiber misalignment; effects of shear and transverse loads on static kinking; viscoelastic and creep kinking; kinking fatigue; and a phenomenological theory of kinking "toughness".

INTRODUCTION

Kinking in fiber composites is a buckling process. As in so many other buckling problems, an adequate understanding of compressive kinking requires more than the traditional ideas of elastic stability and bifurcation of equilibrium paths. *Plasticity* and *imperfection-sensitivity* are involved in an essential way in the kinking of real composites, and *limit-point* buckling, in which a smooth maximum in the applied load is attained, rather than bifurcation buckling, is the rule.

The analysis of kinking in aligned-fiber composites was launched by Rosen (1965), who contemplated the elastic bifurcation buckling of an axially compressed continuum containing perfectly aligned fibers. The widely quoted Rosen result for the critical kinking stress is essentially

$$\sigma_c = G \tag{1}$$

where G is the effective longitudinal shear modulus of the composite. This formula follows directly from the overall moment equilibrium of the kinked configuration sketched in Fig 1. At the buckling stress σ_c, a kink band oriented at a right angle to the loading direction suffers an average shear strain γ equal to the small rotation Φ, and the associated shear stress τ is given by the elastic law $\tau = G\gamma = G\Phi$. The sharp boundary between the kinked and unkinked region is consistent with the assumption of negligible fiber bending

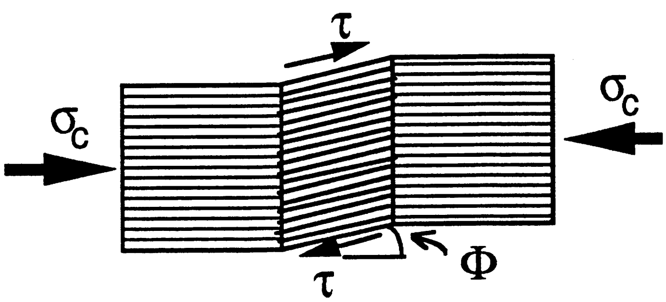

FIG 1. Kink band, normal to fiber direction.

stiffness. (Actually, Rosen's development was somewhat different, and also, he wrote that $\sigma_c = G_m/(1\text{-}c)$, where G_m is the matrix shear modulus, and c is the volume concentration of fibers; but this clearly was meant be interpreted as an approximation to the shear modulus G of a composite in which the fibers are much more stiff than the matrix.)

ASME Reprint No AMR146

It is now well established experimentally (see Budiansky and Fleck 1993 for a review of data) that Eq 1 usually overestimates the actual compressive strength of a polymer-matrix composite substantially, often by a factor of four or so, but the data has a lot of scatter. This scatter and the unreliably high critical stresses based on bifurcation buckling are typical consequences of imperfection-sensitivity. In compressive kinking, however, imperfections identified with initial misalignments of the fibers, are not enough to produce the large observed knockdown factors without the intervention of plastic, or nonlinear, shear deformations. Plasticity and misalignment were both considered by Argon (1973), who announced the simple approximate result

$$\sigma_C = \tau_Y / \bar{\phi} \tag{2}$$

for the kinking stress. Here τ_Y is the shearing yield stress of the composite and $\bar{\phi}$ is the initial maximum angular misalignment of the fibers. This formula can also be deduced from Fig 1 if we now assume the composite material to be rigid-ideally plastic in shear, i.e., incapable of deformation until τ reaches τ_Y, which it can then never exceed. If we say that the rotation Φ in the kink band is $(\bar{\phi}+\phi)$, where $\phi=\gamma$ is an *additional* composite rotation, and set $\tau = \tau_Y$, then moment equilibrium gives (2) as the largest possible value of the compressive stress. This critical stress corresponds to the onset of deformation when ϕ begins to increase from zero. Argon's equation, as well as Rosen's, ignores the fact that kink band orientations are generally observed to be inclined to the transverse direction. In both cases, inclined kink bands would imply somewhat higher critical stresses.

Despite the fact that the Rosen formula (1) is in serious disagreement with experiment, the impression has persisted in some quarters that the compressive strength of polymer-matrix composites is governed primarily by the composite shear modulus. The Argon formula (2), on the other hand, indicates that yield strength and fiber misalignment are the most important parameters. In the rest of this review we will show how these two idealized approaches to static kinking have been unified and extended to cover strain-hardening, inclined kinks, and combined external loading. We will also exhibit theoretical results for the estimation of kink band widths, based on a couple-stress theory that does take into account bending stiffness; show some results for kinking strength based on a stochastic analysis of the influence of randomly distributed misalignments; present results for viscoelastic and creep kinking; mention some work on kinking fatigue; and provide a glance at an engineering theory of kinking "toughness" for the description and control of kink propagation.

STRAIN HARDENING; INCLINED KINKS

If we simply assume the elastic-ideally plastic constitutive law $\tau=G\gamma$ for $\gamma<\gamma_Y$, $\tau=\tau_Y$ for $\gamma>\gamma_Y$, where $\gamma_Y=\tau_Y/G$, the Rosen and Argon results are both encompassed by the relation

$$\sigma_C = \tau_Y / (\gamma_Y + \bar{\phi}) = G/(1+\bar{\phi}/\gamma_Y) \tag{3}$$

(Budiansky 1983) that follows from equilibrium of the normal kink, and which gives (1) for $\bar{\phi}=0$, and approaches (2) for $\bar{\phi}/\gamma_Y$ large. This was further generalized by Budiansky and Fleck (1993) for materials obeying the Ramberg-Osgood shear stress-strain relation

$$\gamma/\gamma_Y = \tau/\tau_Y + (3/7)(\tau/\tau_Y)^n \tag{4}$$

for which limit-point buckling occurs at

$$\sigma_c / G = \left[1 + n\left(\tfrac{3}{7}\right)^{\frac{1}{n}} \left(\frac{\bar{\phi}/\gamma_Y}{n-1} \right)^{\frac{n-1}{n}} \right]^{-1}. \tag{5}$$

It is remarkable that Eq (5) provides nearly the same variation of σ_c/G with $\bar{\phi}/\gamma_Y$ for all values of n in $3<n<\infty$. Values of $\bar{\phi}/\gamma_Y$ in the range 3–4 correspond to σ_c/G around 1/4 to 1/5; for $\gamma_Y=.01$, this implies initial misalignment angles in the vicinity of 2°.

What about inclined kinks? And why do they want to be inclined, as in Fig 2? The kink shown in Fig 2 will develop tension normal to the fibers as a consequence of the transverse strain induced by kink-band rotation for $\beta>0$. A

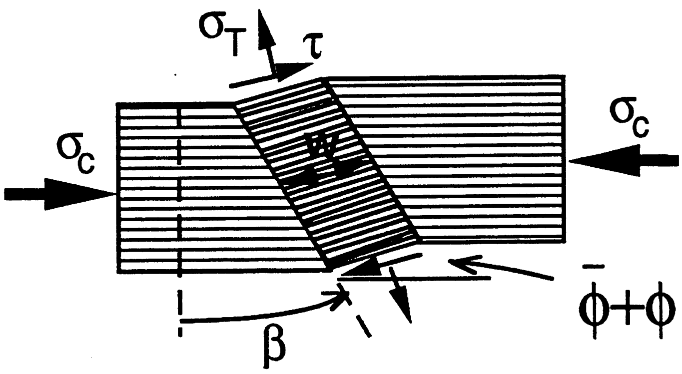

FIG 2. Inclined kink.

primitive, somewhat arbitrary postulated plasticity law for combined transverse tension σ_T and shear τ, in conjunction with static and kinematic analysis of the inclined-kink configuration, provides the interesting result that Eq (5) for the kinking stress remains valid, provided that G and γ_Y are replaced by $G^*=\alpha^2 G$ and $\gamma_Y^* = \gamma_Y/\alpha$, where

$$\alpha = \sqrt{1+(\sigma_{TY}/\tau_Y)^2 \tan^2 \beta} \tag{6}$$

Here σ_{TY} is the plane-strain yield stress in pure transverse tension, analogous to the yield stress τ_Y in Eq (4) for pure shear. In this formulation it is assumed that the fibers are rigid longitudinally (even though they do not resist bending!) so that the composite does not undergo axial straining, and, accordingly, the plasticity is uninfluenced by the longitudinal stress. For kink angles around 20°, misalignment angles inferred from Eq (5) for $\sigma_c / G=1/5$ are only about 3°.

If the assumption of rigid fibers is relaxed, the matrix material in a composite containing perfectly aligned fibers could suffer plastic deformation before kinking occurs, and the Rosen result (1) for elastic bifurcation would have to be replaced by a *plastic* buckling stress, which could be appreciably lower. But calculations by Budiansky and Fleck (1993) for polymer matrix composites show that such reductions would be quite insufficient to account for experimental data; and that in the presence of realistic imperfections in fiber alignment, compressible and rigid fibers lead to nearly the same results for kinking strength.

The first-order effects on the kinking stress σ_c of changes $\delta\tau_Y$ in the shear yield stress and δG in the shear modulus are given by the perturbation equation

$$\frac{\delta\sigma_c}{\sigma_c}=\left[\frac{1}{n}+\left(\frac{n-1}{n}\right)\frac{\sigma_c}{G}\right]\frac{\delta G}{G}+\left(\frac{n-1}{n}\right)\left[1-\frac{\sigma_c}{G}\right]\frac{\delta\tau_Y}{\tau_Y} \qquad (7)$$

that follows from Eq (5). For $\sigma_c / G=1/4$, and $n=\infty$, a given fractional increase in τ_Y is three times as effective as one in G; for n=3, the effects are the same, but it is usually easier to modify the yield stress of a material than its modulus.

All other parameters being equal, the theoretical kinking stress is definitely lowest for $\beta=0$, and so it seems paradoxical that observed kink angles tend o be bounded well away from zero. A tentative explanation was given by Budiansky (1983), who suggested that localized edge misalignments of the fibers would grow into inclined bands of rotation under axial load; and that the directions of these pre-kinking bands would set the size of β. Calculations of these directions based on an *elastic* continuum theory that incorporates the effects of fiber bending resistance suggest that β would like to be in the range of 10° to 35°, depending on the shape of the initial edge imperfection and the elastic constant of the composite.

Kyriakides et al (1993) describe extensive experiments and finite element calculations for the compressive response of finite-width specimens. The calculations incorporate plasticity and a variety of imperfection patterns into a multilayer model for the composite, consisting of alternating lamina of fiber and matrix material. The results provide striking confirmation of the imperfection-sensitivity of the composite strength. Further, the computed kinking response associated with localized edge imperfections tends to support the Budiansky speculation concerning the genesis of the kink angle β. Kyriakides et al find values of β around 8°, somewhat lower than those observed, and dependent on the specimen width.

COMBINED-STRESS LOADING

Consider how the kinking compressive stress would be affected by the presence of an externally applied shear stress τ_c (Batdorf and Ko, 1987; Fleck and Budiansky, 1991; Budiansky and Fleck, 1993). For the case of an elastic-ideally plastic composite, we must keep $\tau_c<\tau_Y$; then, for β = 0, σ_c is still given by Eq (3) if we replace $\bar{\phi}$ by an amplified, effective initial imperfection of magnitude $\bar{\phi}/(1-\tau_c/\tau_Y)$. For strain-hardening, and for $\beta>0$, simple formulas are not available, but numerical calculations by Budiansky and Fleck show that the effects of applied shear stress are less severe. Experiments by Jelf and Fleck (1993) on composite tubes subjected to combined compression and torsion gave results for kinking failure in good agreement with the theoretical predictions based on misalignments of the order of 2°. The additional weakening effects of initial transverse tension have been studied by Slaughter et al (1993); generally, they have less effect than shear stress on the kinking behavior.

KINK WIDTHS

Fully developed kink bands are bounded by fiber breaks, and it is of some interest to see whether the associated kink width w (Fig 2) can be predicted analytically. An early analysis (Budiansky 1983) based on the simplifying assumptions of perfectly aligned fibers and rigid-ideally plastic behavior of the composite in shear and transverse tension, together with incorporation of the effects of couple-stresses provided by the fiber bending, gave

$$\frac{w}{d}=\frac{\pi}{4}\left(\frac{E}{2\tau_Y^*}\right)^{1/3} \qquad (8)$$

for the ratio of the final kink width to the fiber diameter d, in terms of the longitudinal composite modulus E and the β-modified shear yield stress $\tau_Y^*=\alpha\tau_Y$. This formula was based on the additional assumption that the fibers were perfectly brittle in tension. Measurements of kink-band widths by Jelf and Fleck (1992) were in fairly good agreement with Eq (8) over a wide range of parameters. An analytic generalization was derived by Fleck et al (1993), who considered an elastic-ideally plastic composite, and also took finite values of fiber failure strain ε_F into account. They present the implicit equation

$$4\left(\tau_Y^*/E\right)^2\left[\left(\frac{4w}{\pi d}\right)^2-4\gamma_Y^*\left(\tau_Y^*/E\right)^{-1}\right]=\left[\varepsilon_F+\left(\frac{4w}{\pi d}\right)^{-2}\right]^2 \qquad (9)$$

connecting w/d to τ_Y^*/E, γ_Y^*, and ε_F. For reasonable values of these parameters (say, .0005, .01, and .01), Eq (8) gives w/d ≈ 12.

Fleck et al also give numerically calculated estimates of w/d for strain-hardening materials. Typical results are shown in Fig 3 for several values of the Ramberg-Osgood index n. In the calculation of these results, fiber rotation begins at the peak Rosen stress (1) (with G replaced by G*), and the fiber fractures bounding the kink band occur during the subsequent

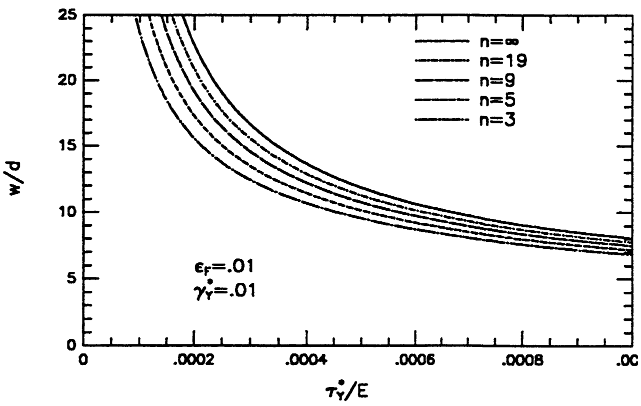

FIG 3. Kink width; effect of strain-hardening parameter n.

drop in load.. Repetition of the calculations with an initial misalignment pattern taken into account showed that although the peak stress is reduced, the final kink band width remains about the same as it was for the perfect composite. Furthermore, over a wide range of parameters, the peak stress, now calculated with bending resistance included, is still approximated well by Eq (5), when $\bar{\phi}$ is taken as the maximum misalignment angle in the assumed imperfection shape. Only for unrealistically short wavelengths of initial imperfection will the simple formula (5) give results that are too low by more than negligible amounts.

RANDOM MISALIGNMENTS

An initial exploration of the effects of random distributions of fiber inclinations has been made by Slaughter and Fleck (1994). Briefly, the plan was to assume a flat power spectral density for misalignment angle as a function of distance in the fiber direction (with perfect correlation along inclined β-lines transverse to the fibers),with a lower cut-off for the spectral wavelength, and with an assigned value for the mean-square fiber slope. Monte-Carlo realizations for the fiber waviness were generated, for each of which peak composite compressive stresses σ' were calculated, with couple-stress theory used to take bending resistance into account. These results were then used to compute a probability density for the ratio σ'/σ_c, where σ_c is given by (5), with $\bar{\phi}$ chosen as the maximum inclination in each realization.

We can not go into the details here, but as expected, the results show that (5) loses high reliability when the cut-off wavelength is low enough. However, assuming a flat distribution for misalignment wavelength may bias the results unduly with respect to the importance of small-wavelength imperfections. This subject merits further study; in particular, real imperfection spectra should be measured.

VISCOELASTIC AND CREEP KINKING

Time-dependent compressive response has been studied by Slaughter and Fleck (1993a) for the case of linear viscoelastic behavior in shear and transverse tension, and by Slaughter et al (1993) for nonlinear creep. We summarize the latter here, in a slightly modified form.

The three-parameter constitutive equation

$$\dot{\gamma}/\dot{\gamma}_{ref} = \left(\tau/\tau_{ref}\right)^M \qquad (10)$$

for shear strain rate in terms of shear stress, which ignores time-independent plasticity effects, is a starting point for the formulation of generalized equations for combined shear and transverse tension, akin to those used in the earlier static problem. Here $\dot{\gamma}_{ref}$ is a reference value of creep rate produced by the reference shear stress τ_{ref}. Incorporating the constitutive law into the equilibrium analysis of Fig 2 then gives the relation

$$t = \left(\frac{1}{M-1}\right)\left(\frac{1}{\dot{\gamma}^*_{ref}}\right)\left(\frac{\tau^*_{ref}}{\sigma}\right)^M\left[\frac{1}{\bar{\phi}^{M-1}} - \frac{1}{\left(\bar{\phi}+\phi\right)^{M-1}}\right] \qquad (11)$$

between the time t, the applied compressive stress σ, and the additional rotation ϕ. Here the kink angle is taken into account through the definitions

$$\dot{\gamma}^*_{ref} = \dot{\gamma}_{ref}/\alpha, \quad \tau^*_{ref} = \alpha\tau_{ref}, \quad \alpha = \sqrt{1+R^2\tan^2\beta} \qquad (12)$$

where (see Eq (6)) R can be regarded as a parameter relating transverse creep strength to shear creep strength. We can now set a criterion for the creep kinking lifetime t_f in one of several ways: a critical value ϕ_f can be assigned, corresponding to tensile fracture at the fiber-matrix interface; or we can say that static plastic kinking occurs at the time when $\bar{\phi}+\phi$ reaches a value of misalignment that, together with $\sigma_c=\sigma$, satisfies the static criterion (5); or we can simply say that an upper bound approximation to t_f corresponds to $\phi=\infty$ in (11). This last criterion is the simplest, and gives

$$t_f \approx \left[(M-1)\dot{\gamma}^*_{ref}\bar{\phi}^{M-1}\left(\sigma/\tau^*_{ref}\right)^M\right]^{-1}. \qquad (13)$$

It is of some interest to see whether creep kinking might be an issue in ceramic fiber/metal matrix composites, under conditions of moderately elevated temperature and sustained high load. Assuming the plausible values $\tau_{ref} = 100\,\text{MPa}$, $\dot{\gamma}_{ref} = 10^{-7}\,\text{sec}^{-1}$, σ=1500 MPa, $\bar{\phi} = 3°$, and M = 5, gives $t_f \approx 120$ hours, which suggests that creep kinking may indeed have to be considered in design of metal-matrix composites.

KINKING FATIGUE

Slaughter and Fleck (1993b) have analyzed fatigue kinking from two viewpoints: (i) fatigue failure by low cycle fatigue of the matrix within the kink band, and (ii) failure by cyclic ratchetting of the material within the kink band until the plastic strain accumulation is sufficient to precipitate the plastic microbuckling instability. Little experimental data are available on compressive fatigue of fiber composites. Soutis et al (1991) observed fatigue kink growth from a notch in a carbon fiber epoxy composite, and Huang and Wang (1989) measured the stress-life fatigue curve for unnotched specimens made from alumina fibers in an aluminum alloy matrix. Slaughter and Fleck (1993b) found that the predictions of the ratchetting fatigue model were in better agreement with the experimental results of Huang and Wang than the predictions of the low cycle fatigue model. Further work is required to elucidate the fatigue failure mechanisms as a function of material composition.

KINKING TOUGHNESS

A novel engineering model has been developed by Fleck and co-workers (Soutis, Fleck and Smith, 1991; Soutis, Curtis and Fleck, 1993; Sutcliffe and Fleck, 1993) to predict the compressive kinking failure of laminated carbon fiber epoxy panels containing a single hole. The crack model is of the large-scale bridging type and assumes that a kink band emanating from the hole behaves like an *overlapping mode I crack*, with normal crack bridging compressive stresses that drop linearly with crack overlap from a maximum value of the unnotched compressive strength. The area under the curve of crack bridging stress versus overlap displacement is derived from a separate compressive kink propagation experiment, wherein the "toughness" of a specimen containing a sharpened long slit is measured. This simple crack bridging model gives an accurate prediction of failure load and critical kink length at failure for a range of hole sizes and laminates (Soutis, Curtis and Fleck, 1993). Further support for the notion that the microbuckled band behaves like a mode I crack comes from the observation that the overlapping displacement behind a growing kink band increases with increasing distance behind the kink-band tip in the manner predicted by a crack bridging model (Sutcliffe and Fleck, 1994)

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