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Fatigue life prediction of a structural steel under service loading

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Fatigue life prediction techniques for variable amplitude load histories are reviewed. The fatigue crack growth rate and crack closure responses of BS4360 50B steel are determined for a service load history experienced by a gas storage vessel. Crack propagation rates are found to be independent of specimen thickness. Crack growth is successfully predicted by linear summation using the Paris law; no significant improvement is achieved by incorporating crack closure into the analysis. The particular choice of cycle counting technique is also found to have an insignificant effect on the predicted fatigue life. The load-interaction model proposed by Willenborg *et al* correctly indicates the absence of retarded growth, whilst the Wheeler and Führing models erroneously predict retarded crack growth.

Key words: fatigue; fatigue crack closure; fatigue life prediction; variable amplitude fatigue; BS4360 50B steel; compliance measurement

High-pressure gas pipelines and storage vessels suffer a daily pressure fluctuation due to variations in supply and demand. The pressure vessels are charged to an almost constant maximum pressure overnight and discharged during the day to varying minimum pressures (see Fig. 1). Cracks which exist in the welded joints of these vessels are therefore subjected to variable amplitude loading proportional to this pressure history.

Recent work[1] has shown that variable amplitude waveforms of constant K_{max} and varying K_{min} cause long fatigue cracks to grow at accelerated rates compared with a linear summation of damage. The object of this paper is to compare the measured crack growth response of a structural steel subjected to a service load history with the response predicted by a number of crack growth models. Fatigue crack growth is investigated for both plane stress and plane strain conditions.

Review of fatigue crack growth prediction methods for variable amplitude loading

Prediction methods for fatigue crack propagation fall into two distinct categories, the characteristic approach and the cycle-by-cycle approach.

The characteristic approach

The characteristic approach assumes that the relationship between crack growth rate, da/dN, and a characteristic stress intensity range of the load history, ΔK_{char}, is preserved for several types of loading, including constant amplitude. For example, the root-mean-square stress intensity range, ΔK_{RMS}, has been used successfully by Barsom[2] to correlate fatigue crack growth in steels under bridge-type service load spectra.

Similarly, Dover[3] has used the ΔK_{RMS} parameter to predict crack growth rates in offshore structures subjected to random wave loading.

Schijve,[4] however, has met with difficulties in predicting crack growth rates using a characteristic ΔK. He subjected 2024-T3 Alclad sheet to random and statistically equivalent programmed loading. The waveforms were based on a flight-simulation gust loading spectrum. Schijve found that the random history produced shorter fatigue lives than the programmed loading, for the same values of ΔK_{char}. Clearly, the characteristic approach is no more than an empirical tool and has no physical basis: it does not attempt to predict load-interaction effects using any physical model, and is considered no further in the present study.

The cycle-by-cycle approach

The cycle-by-cycle approach sums the crack advance associated with each load cycle in order to predict the overall fatigue life of a structure. The incremental crack advance associated with each load cycle may be assumed to equal the crack growth rate associated with constant amplitude loading of the same magnitude, or it may be deduced from a load-interaction model. Different load-interaction models may be used, depending upon the particular physical mechanism causing the interaction effects. For instance, the models of Wheeler[5] and Willenborg *et al*[6] assume that overloads lead to high compressive residual stresses ahead of the crack tip and thereby to crack growth retardation. Alternatively, the Führing model,[7,8] amongst others, assumes that accelerated and retarded growth is due to the crack closure phenomenon.

When using the cycle-by-cycle approach, we must define what is meant by a fatigue cycle. Several cycle counting methods are used in fatigue crack initiation and propagation studies. Currently, range-mean, rainflow and level-crossing counting are the most popular. A description of these techniques is given by Fuchs and Stephens[9] and Broek.[10] Socie[11] has shown that the rainflow method successfully accounts for the plastic deformation history at the tip of a fatigue crack and thus argues that it is the most appropriate method. Yet, many crack growth models, such as those of Wheeler and Willenborg *et al*, assume range-mean counting, and in the aerospace industry the level-crossing method is often employed.[10]

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Currently, there is no consensus on the best load-interaction model and cycle-counting procedure, but a growing body of experimental evidence[11–15] suggests that crack closure arguments and the rainflow method give the most accurate predictions of fatigue life.

Crack closure arguments

Experiments show that the crack opening load varies cycle-by-cycle when the block length of a repeated load history is long,[16] but remains constant when the block length is short.[11–18] Elber[17] has defined a long load history as one which repeats itself after a crack growth increment of greater than the forward plastic zone size; these long load sequences are termed non-stationary.[19] Alternatively, when the variable amplitude load history is repeated exactly and regularly, the waveform is stationary.[19]

The closure-based crack growth models of Führing[7,8] and de Koning[20] predict a continually varying crack opening load; hence, they are useful only for calculating crack growth under non-stationary loadings. Newman's numerical crack closure model[21] predicts a change in crack opening load after fixed crack growth increments rather than after each load cycle, and so can be used for both stationary and non-stationary load histories.

The pressure vessel load history illustrated in Fig. 1 is stationary in nature (see results, below). Previous work on such load histories is now reviewed.

Fatigue life prediction for stationary loading

Socie[11] and Kikukawa and co-workers[12,13,15] have recently examined crack growth in steels of varying strength, using short, repeated load histories. Kikukawa *et al*[14] also examined ZK141-T7 aluminium alloy. Socie employed the three standard SAE automotive histories,[22] while Kikukawa and co-workers[12–15] used pseudo-random, periodic underloads, block loading and other simple program loading. In each case, fatigue lives were successfully estimated to within a factor of about 2, using the following assumptions:

1) the crack opening stress intensity, K_{op}, is constant during each program block of the stationary, variable amplitude loading and is fixed by the maximum stress intensity, $(K_{max})_{VA}$, and the minimum stress intensity, $(K_{min})_{VA}$, which occurs in each program block. Further, the crack opening load, K_{op}, is assumed to equal the crack opening load in a constant amplitude test of maximum stress intensity, $K_{max} = (K_{max})_{VA}$, and minimum stress intensity, $K_{min} = (K_{min})_{VA}$. This assumption is reasonable because:

- K_{op} depends directly upon crack tip plasticity when growth rates are above threshold;
- the maximum forward plastic zone sizes for the constant and variable amplitude loadings are equal for $K_{max} = (K_{max})_{VA}$; and
- the maximum reversed plastic zone sizes for the constant and variable amplitude loadings are equal for $(K_{max} - K_{min}) = ((K_{max})_{VA} - (K_{min})_{VA})$.

2) crack growth is integrated on a cycle-by-cycle basis, using the modified Paris law:

$$\frac{da}{dN} = C'(\Delta K_{eff})^{n'} \text{ mm/cycle} \tag{1}$$

where da/dN is the crack growth increment accompanying an effective stress intensity cycle, ΔK_{eff}, and C' and n' are best-fitting constants from constant amplitude $da/dN - \Delta K_{eff}$ data. We define ΔK_{eff} in the usual way:

$$\left.\begin{aligned} \Delta K_{eff} &= K_{max} - K_{min} \quad \text{for } K_{min} \geqslant K_{op} \\ &= K_{max} - K_{op} \quad \text{for } K_{min} < K_{op} \end{aligned}\right\} \tag{2}$$

where K_{max} and K_{min} are the maximum and minimum stress intensities of the current load cycle.

3) cycles are counted using the rainflow method,[23] or a similar method known as range-pair counting.[9]

Schijve has successfully predicted crack growth rates in 2024-T3 aluminium alloy under flight simulation loading using assumptions 1 and 2, together with the range-mean counting technique. However, it is clear from the work of Socie[11] and Kikukawa and co-workers[12–14] that the rainflow method is more accurate than simple range-mean counting by up to a factor of 8 on life.

Assumption 1 has been substantiated by measurements of the crack opening load during variable amplitude loading.[11–15,17] Since we can deduce the crack opening behaviour due to variable amplitude loading directly from the constant amplitude response, complex closure models are not required for an estimation of K_{op}.

Recently, doubt has been cast on the validity of the cycle-by-cycle approach for fatigue life prediction. Whilst Hertzberg and Paris[24] have been able to find a one-to-one correspondence between macroscopic crack growth rates and striation spacings in 2024-T3 aluminium alloy, no such correlations have been found for 7075-T651 aluminium alloy and steels in other, more recent, investigations;[25–27] rather, many load cycles were required to advance the crack by each increment. Thus, doubt is cast on the physical basis of Equation (1) and, more generally, on the cycle-by-cycle approach. At present, there is no more accurate method for fatigue life predictions, and so the cycle-by-cycle approach remains the most popular.

Experimental details

The rate at which fatigue cracks grow in BS4360 50B steel specimens was determined for both plane stress and plane strain conditions, using the pressure vessel load history depicted in Fig. 1. This history was the only actual service

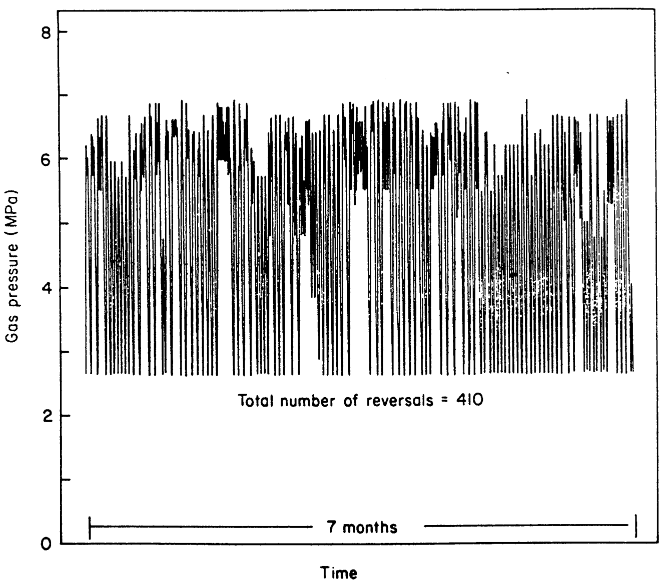

Fig. 1 Actual service load history experienced by a gas pressure vessel

loading available for which load-sequence effects were expected. It is untypical in that it was recorded during the commissioning phase of a compressor station. Gas transmission pipeline and storage vessels will never be subjected to such severe pressure fluctuations: therefore this load history represents the worst case.

Compact tension specimens, of width 50 mm and thicknesses (B) of 3 and 24 mm, were cut from a single 25 mm plate of hot-rolled BS4360 50B steel, such that cracks grew transverse to the rolling direction. The 0.14% C, 1.27% Mn steel had a yield stress of 352 MPa and ultimate tensile stress of 519 MPa in the rolling direction. The specimens were stress-relieved for 1 h at 650°C prior to testing.

Fatigue tests were conducted using a conventional servo-hydraulic test-machine; the test-machine was operated in load-control and driven by a novel microcomputer system.[28] The magnitude of the loading was chosen to give crack growth rates consistent with a 40-year service life. A constant loading rate was used such that the frequency of the largest cycle in the load spectrum was 2 Hz while the frequency of the smallest cycle was 23 Hz. No load-shedding technique was employed.

Crack growth was monitored using the DC potential drop method, and crack opening loads were recorded with both a crack mouth displacement and a back-face strain gauge. Additionally, a twin cantilever clip gauge was mounted near the crack tip, for measuring the closure response of the thin specimen. Discrimination of the crack opening load was improved through use of third order low-pass filters of 1 Hz cut-off frequency, and by use of an offset procedure (see Fig. 2). Load/displacement and load/offset displacement closure traces were taken at a reduced test frequency of 0.01–0.1 Hz, to avoid signal distortion due to the low-pass filters. The crack opening load was defined as the point where the load/displacement trace (and therefore load/offset displacement trace) first became linear (see Fig. 2). Full details of the potential drop and closure measurement system are given in References 27 and 29.

Results

The intensity of the loading was such that the calculated plane stress forward plastic zone size lay in the range 0.5–5 mm. Thus, the thin specimen (B = 3 mm) was in a state of plane stress at the crack tip, while the thick specimen (B = 24 mm) suffered plane strain deformations.

The influence of thickness upon crack growth response is shown in Fig. 3, where crack growth rate is plotted against the characteristic stress intensity range, ΔK_{char}, defined by:

$$\Delta K_{char} = (K_{max})_{VA} - (K_{min})_{VA} \quad (3)$$

Here, $(K_{max})_{VA}$ is the maximum stress intensity in a program block, and $(K_{min})_{VA}$ is the minimum stress intensity in a program block. Clearly, for the service load history considered, cracks grew at the same rate under both plane stress and plane strain conditions. The crack growth rate per loading block was always much less than the forward plastic zone size and so we can consider the load history as stationary in character, by using the definition of Elber.[17]

The measured crack opening load was found to be independent of the type of closure gauge, though the compliance traces associated with the crack-mouth gauge

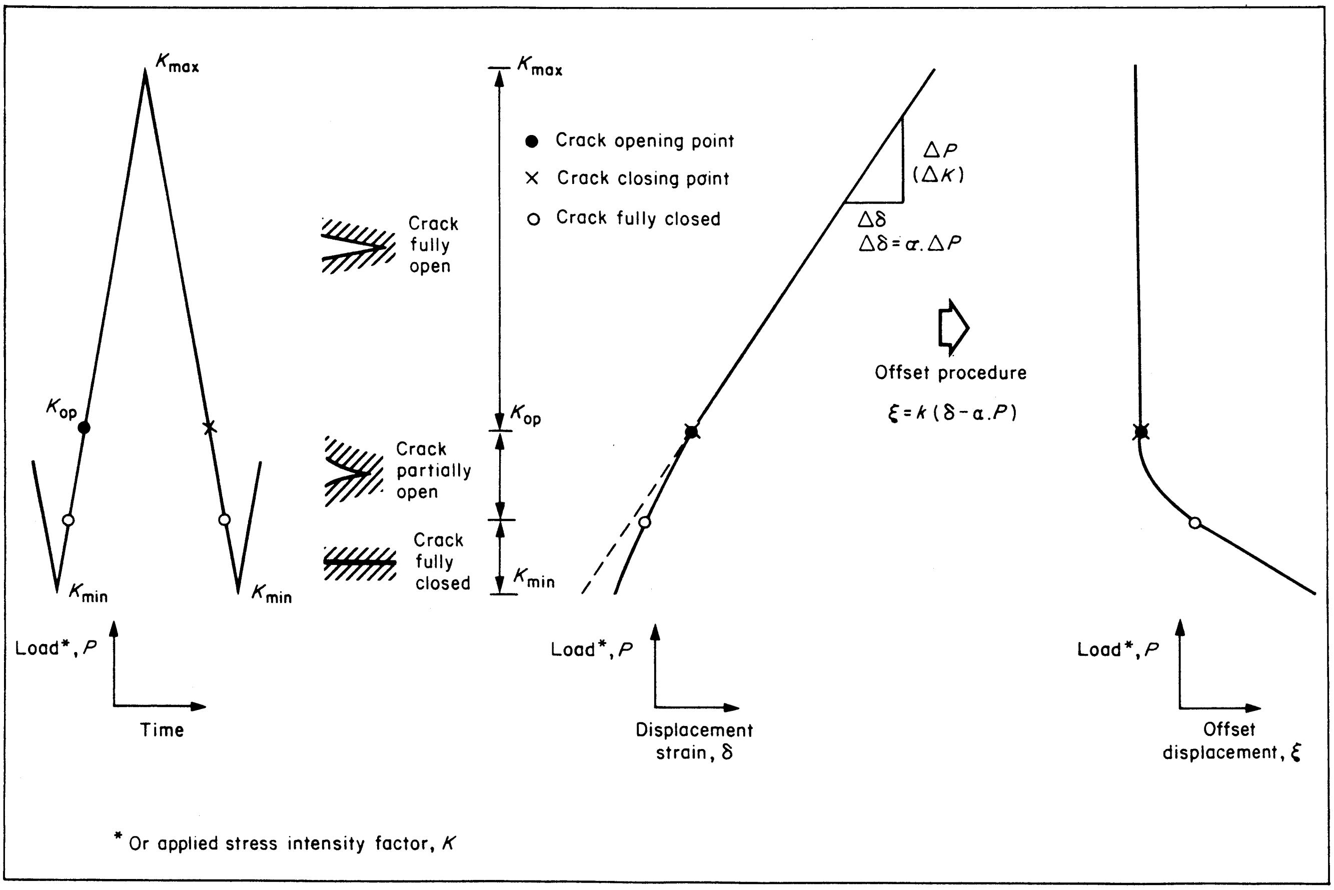

Fig. 2 Effect of closure of fatigue crack upon specimen compliance — idealized response

were difficult to interpret due to excessive hysteresis. Also, the crack opening level, K_{op}, was constant from cycle to cycle within a program block.

We may define a closure value $(U)_{VA}$ for variable amplitude loading in terms of $(K_{max})_{VA}$, $(K_{min})_{VA}$ and K_{op}:

$$(U)_{VA} = \frac{(K_{max})_{VA} - K_{op}}{(K_{max})_{VA} - (K_{min})_{VA}} \quad (4)$$

The closure responses of the 3 mm thick and 24 mm thick specimens of BS4360 50B steel are compared in Fig. 4 with previous results for the same type of specimens under constant amplitude loading.[27] The load ratio $((K_{min})_{VA}/(K_{max})_{VA})$ for the service load history was 0.35 while the load ratio (K_{min}/K_{max}) for the constant amplitude loading was 0.30. It is apparent from Fig. 4 that the closure response is independent of the type of load history provided we consider similar load ratios and put $K_{max} = (K_{max})_{VA}$ and $K_{min} = (K_{min})_{VA}$. Further, the closure values $(U)_{VA}$ and U increase with increasing $\Delta K_{char}((K_{max})_{VA} - (K_{min})_{VA})$ and ΔK, respectively. Such decreases in the amount of closure with increasing crack tip loading are probably due to a small specimen size in comparison with the crack tip forward plastic zone size.[27]

We also note from Fig. 4 that the 24 mm thick specimens show 10% less closure than the 3 mm thick specimens. The lower crack opening loads in the thick specimens are ascribed to greater plastic constraint at the crack tip.[30]

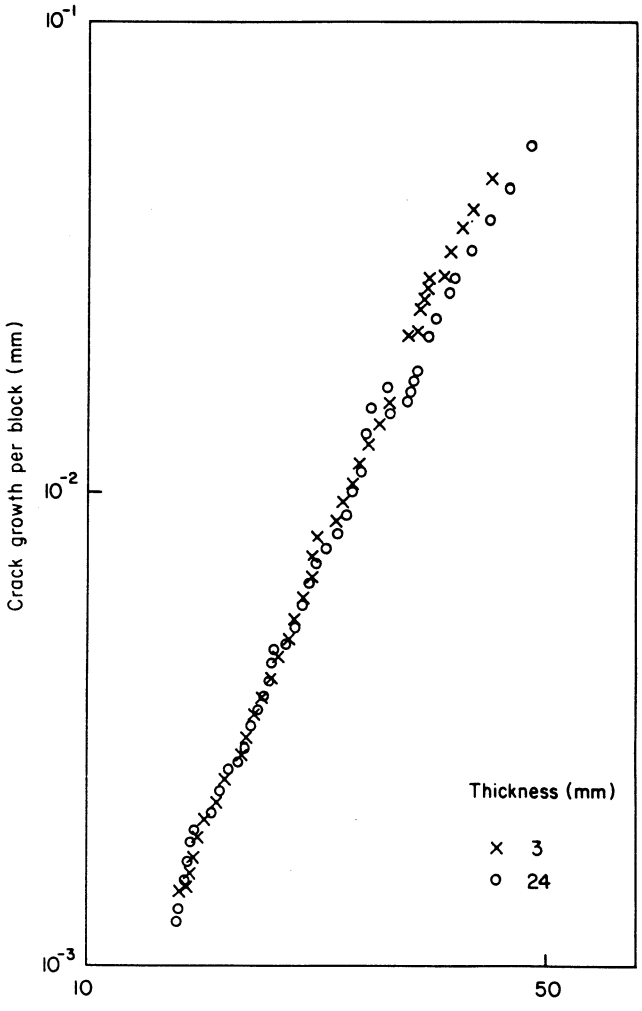

Fig. 3 Effect of thickness upon crack growth response of BS4360 50B steel under service load history: $\Delta K_{char} = (K_{max})_{VA} - (K_{min})_{VA}$, where $(K_{max})_{VA}$ is the maximum stress intensity in a program block and $(K_{min})_{VA}$ is the minimum stress intensity in a program block

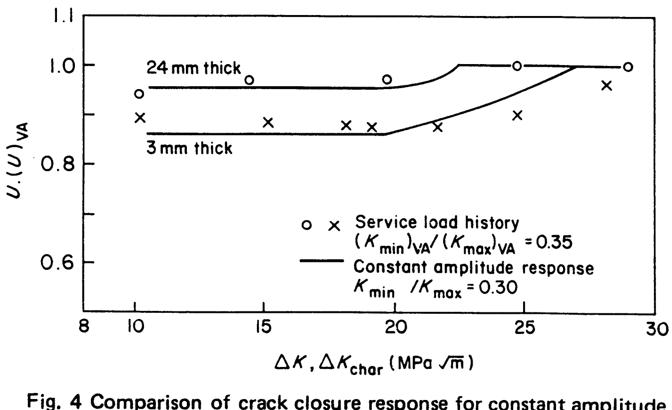

Fig. 4 Comparison of crack closure response for constant amplitude and service load histories: $\Delta K = K_{max} - K_{min}$, $\Delta K_{char} = (K_{max})_{VA} - (K_{min})_{VA}$

Prediction of crack growth rates

Crack growth rates were predicted using a number of crack growth models and compared with the measured response of Fig. 3. The models fall into two groups: linear integration of growth and non-linear models capable of accounting for load-interaction effects.

Linear integration of growth

Since the pressure vessel load history is stationary and the crack opening stress intensity, K_{op}, is fixed by $(K_{max})_{VA}$ and $(K_{min})_{VA}$ (see Fig. 4) we may predict crack growth rates in the manner used by Socie,[11] Kikukawa and co-workers[12–15] and Schijve.[16] In particular, the three counting methods, rainflow, range-mean and level crossing, are considered. Crack growth was integrated using the modified Paris law:

$$\frac{da}{dN} = 1.07 \times 10^{-8}(\Delta K_{eff})^{2.95} \text{ mm/cycle} \quad (5)$$

where ΔK_{eff} is expressed in units of MPa $\sqrt{m}$. The two numerical constants of Equation (5) were obtained by a least-squares linear regression on constant amplitude crack growth data from 3 mm and 24 mm thick BS4360 50B steel specimens.[27] Equation (5) adequately describes the constant amplitude response for crack growth rates in the range 1.5×10^{-7} mm/cycle to 10^{-3} mm/cycle and a load ratio in the range 0.05 to 0.5 (see Fig. 5). Whilst constant amplitude tests show a definite threshold, $(\Delta K_{eff})_{th}$, of 3.0 ± 0.3 MPa $\sqrt{m}$, we assume $(\Delta K_{eff})_{th}$ is zero in the variable amplitude tests, as found by Kikukawa *et al.*[15]

Crack growth predictions, assuming K_{op} values equal to the constant amplitude response of Fig. 4, are given in Figs 6 and 7. It is clear that all three counting methods, rainflow, range-mean and level crossing, yield accurate estimates of crack growth rates, given the usual scatter in crack growth rates of a factor of 2 to 3. It is not surprising that the three counting techniques yield similar predictions of growth rate, since they reduce the service history into similar distributions of load ranges (see Fig. 8).

The crack growth rate response due to the service load history was also predicted by a straightforward inte-

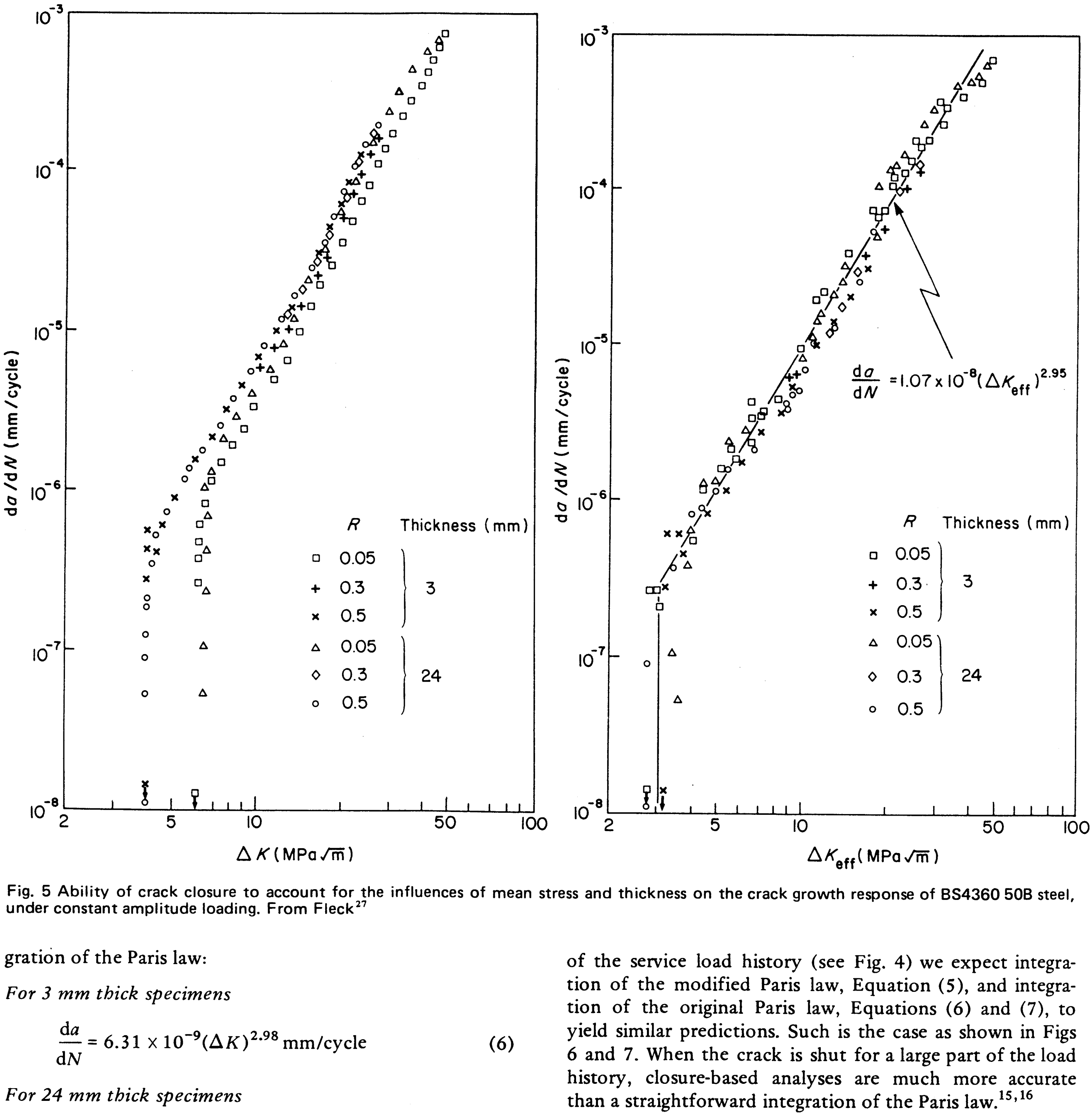

Fig. 5 Ability of crack closure to account for the influences of mean stress and thickness on the crack growth response of BS4360 50B steel, under constant amplitude loading. From Fleck[27]

gration of the Paris law:

For 3 mm thick specimens

$$\frac{da}{dN} = 6.31 \times 10^{-9}(\Delta K)^{2.98} \text{ mm/cycle} \tag{6}$$

For 24 mm thick specimens

$$\frac{da}{dN} = 4.98 \times 10^{-9}(\Delta K)^{3.12} \text{ mm/cycle} \tag{7}$$

where ΔK is expressed in units of MPa $\sqrt{\text{m}}$.

The regressions given by Equations (6) and (7) were calculated from constant amplitude crack growth data, for a load ratio lying in the range 0.3 to 0.5, and crack growth rates in the range 5×10^{-7} mm/cycle to 10^{-3} mm/cycle. Mean stress has an insignificant influence on growth rate for such conditions.[27] As before, it was assumed that the threshold, ΔK_{th}, disappeared on application of variable amplitude loading, so Equations (6) and (7) were assumed to remain valid at growth rates below 5×10^{-7} mm/cycle.

Figs 6 and 7 include predictions of the crack growth rate response by a linear integration of the Paris law and a range-mean count. The resulting predictions are at least as accurate as the closure-based analysis, and are quite adequate for design purposes. Since the crack is held open for much of the service load history (see Fig. 4) we expect integration of the modified Paris law, Equation (5), and integration of the original Paris law, Equations (6) and (7), to yield similar predictions. Such is the case as shown in Figs 6 and 7. When the crack is shut for a large part of the load history, closure-based analyses are much more accurate than a straightforward integration of the Paris law.[15,16]

We conclude that the service load history shown in Fig. 1 gives rise to neither accelerated nor retarded growth in BS4360 50B steel; a simple linear summation of growth with no recourse to crack closure adequately predicts the fatigue response.

Fleck[1] and Fleck and Smith[31] have recently investigated the fatigue response of BS4360 50B steel using simple program loading; tests were performed using K-control, so crack growth rates could be measured very accurately.[28,32] When the simple waveform included in Fig. 9 was employed, cracks grew 10% faster in the 3 mm thick specimens and 20% faster in the 24 mm thick specimens than predicted by a straightforward linear summation of growth based on the Paris law.* The distribution of load ranges in the simplified waveform and in the service load history are compared

*$(U)_{VA} = U = 1$ for these tests, hence the Paris law and modified Paris law result in the same predictions of crack growth rate. The accelerated growth is not due to the crack closure phenomenon.

in Fig. 9, using an exceedance diagram. This exceedance diagram has been generated by the level-crossing method of cycle counting, and is simply a cumulative frequency diagram showing the total number of times that each stress level is exceeded in one program block. The exceedance diagram for the simplified waveform is similar in shape to that of the service load history (see Fig. 9); hence we may expect slightly accelerated growth due to the service loading. Any small accelerations are masked by the scatter associated with tests performed under load control, so we cannot resolve accelerated growth from Figs 6 and 7. Instead, we deduce that models based on a linear integration of damage adequately account for the crack growth response. How much in error, then, are load-interaction models which predict accelerated or retarded growth?

Load-integration crack growth models

We consider three models capable of accounting for load-interaction effects, the models of Willenborg *et al*,[6] Wheeler[5] and Führing.[7,8]

The Willenborg model

The retardation model of Willenborg *et al*[6] is based on the assumption that overloads induce compressive residual stresses ahead of the crack tip and thereby reduce crack tip stress intensities by an amount, K_{red}. The quantity K_{red} is a function of the forward plastic zone size associated with the current load cycle of a load sequence and of the overload plastic zone size due to previous overloads. The model predicts crack growth retardation when the nominal minimum stress intensity of the current load cycle is less than K_{red}. In the case of the pressure vessel load history, K_{min} is greater than K_{red} throughout the load history and so no retardation is predicted. The Willenborg model yields the same crack growth response as a straightforward integration of the Paris law with a range-mean count (see Figs 6 and 7) and is thus successful.

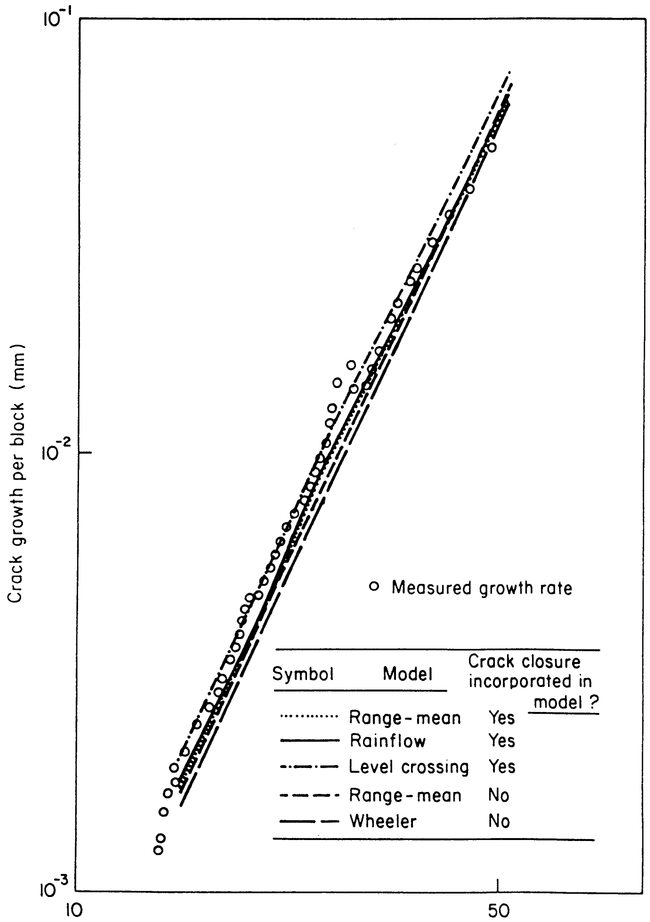

Fig. 6 Comparison of predicted and measured crack growth rates for service load history, 24 mm thick specimens. $\Delta K_{char} = (K_{max})_{VA} - (K_{min})_{VA}$

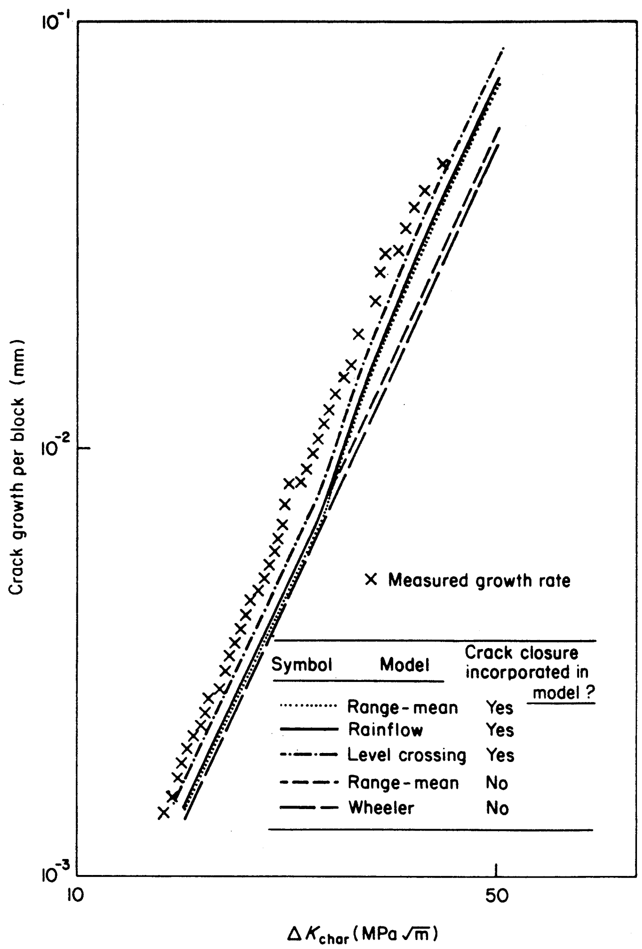

Fig. 7 Comparison of predicted and measured crack growth rates for service load history, 3 mm thick specimens. $\Delta K_{char} = (K_{max})_{VA} - (K_{min})_{VA}$

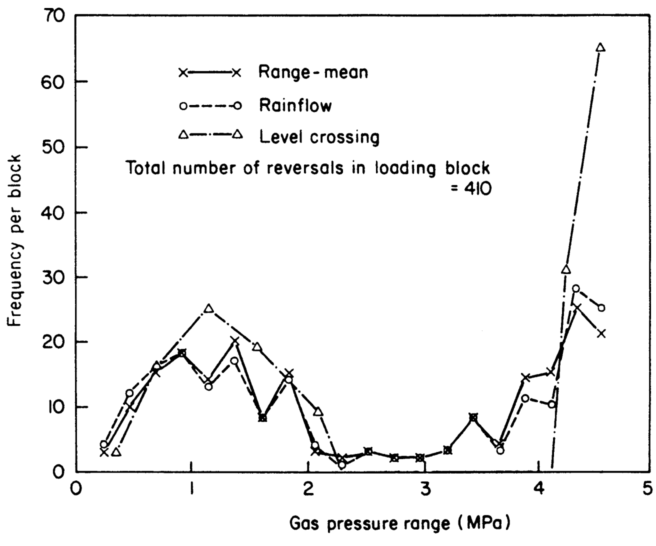

Fig. 8 Analysis of service load history using 3 different counting techniques

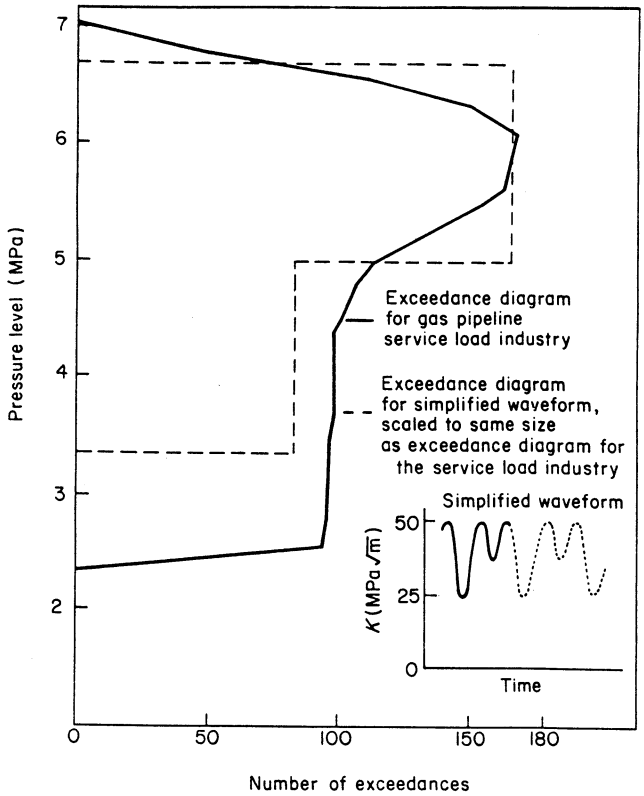

Fig. 9 Comparison of exceedance diagrams for the service load history and for a much simplified waveform

The Wheeler model

Wheeler's retardation model[5] is also based upon the supposition that compressive residual stresses ahead of the crack tip lead to retarded growth. It assumes that the crack growth rate due to the current load cycle of a load sequence is equal to the equivalent constant amplitude growth rate (as given, for example, by Equations (6) and (7)) multiplied by a retardation factor, ϕ. Wheeler defines ϕ as:

$$\phi = \left(\frac{r_p}{d}\right)^m \tag{8}$$

where r_p is the forward plastic zone size associated with the current load cycle and d is the distance from the crack tip to the overload plastic zone boundary due to previous overloads. The parameter m, termed the Wheeler index, is adjusted to give the best overall crack growth predictions for a particular material and class of loading spectra. Previous work on periodic overloads[1] gave a value for m of 0.75 for BS4360 50B steel. The Wheeler model implicitly assumes a range-mean count of the load history, in the same manner as the model of Willenborg *et al.*

Crack growth predictions using the Wheeler model in conjunction with Equations (6) and (7) are included in Figs 6 and 7. The model erroneously predicts retarded growth by a factor of about 2 and, therefore, should not be used for the life prediction of gas pressure vessels.

The Führing model

Führing has recently developed a crack growth model[7,8] based on the crack closure phenomenon. His model is a generalization of the theoretical crack growth response of an elastic-perfectly plastic solid subjected to a single peak overload, and to a step increase in mean load. Führing's model was used to predict the crack growth rate due to the service load history of Fig. 1; it was found that the model gave crack growth rates an order of magnitude slower than those observed. The discrepancy is partly due to the fact that the Führing model predicts that the effective stress intensity range associated with a load history of zero K_{min} and gradually decreasing K_{max} is $0.5\Delta K_i$, where $\Delta K_i (= K_{max} - K_{min})$ is the stress intensity range due to each successive load cycle. Since K_{max} is decreasing gradually from one cycle to the next, retarded growth does not occur and the effective stress intensity range is, in reality, ΔK_i.

There are other drawbacks associated with Führing's approach:

- for constant amplitude loading, his theoretical model[33] predicts much higher crack opening loads than are observed experimentally[27]
- he modelled crack growth retardation by generalizing his theoretical solution for a crack subjected to a single peak overload. In particular, he examined the recovery of crack growth rates from the post-overload minimum growth rate back to the stabilized pre-overload value. Führing found that the constant amplitude relation between crack growth rate, da/dN, and the effective stress intensity range, ΔK_{eff}, is preserved during this recovery period. Fleck *et al*[30] have recently shown that, during this transient period of increasing growth rates, the crack closes in a discontinuous manner, resulting in faster growth than predicted by the constant amplitude $da/dN - \Delta K_{eff}$ relation. The disagreement is due to the fact that Führing assumed that the residual hump of stretched material left in the wake of the crack by the overload acted like a rigid block, whereas Fleck *et al*[30] observed it to behave like a compliant spring.

We conclude that the Führing model is unable to account for the crack growth response of BS4360 50B steel subjected to a gas pressure vessel load history.

The Willenborg, Wheeler and Führing models are all semi-empirical models developed for aircraft materials and load histories; it is therefore not surprising that two of them do not work in the present case.

Conclusions

In this study, BS4360 50B structural steel was subjected to a gas pressure vessel load history. Under these conditions, fatigue cracks grew at a rate predicted by a simple linear summation of growth, using the Paris law. Since the mean stress of the loading was high, the cracks were held open for much of the load history and crack closure considerations led to no improvement in the predicted fatigue life. The choice of cycle counting method had also little influence on the accuracy of fatigue life predictions. For this particular load history there was no retardation, in agreement with the model of Willenborg *et al*, but contrary to the predictions of the Wheeler and Führing models.

It was found that the crack opening load was constant throughout a program block of the service load history, in disagreement with the fundamental assumptions of many theoretical crack closure models. The crack opening load was a function of the maximum overall stress intensity, $(K_{max})_{VA}$, and the minimum overall stress intensity, $(K_{min})_{VA}$, in a program block. Specifically, the crack opening load was equal to that of a constant amplitude waveform of $K_{max} = (K_{max})_{VA}$ and $K_{min} = (K_{min})_{VA}$.

Finally, tests showed that the crack growth rate and closure responses of BS4360 50B steel, subjected to this load history, were little influenced by a change in stress state from plane stress to plane strain.

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