An overview of the mechanical properties of foams and periodic lattice materials

Norman A. Fleck

Cambridge University Engineering Department, Trumpington St.,
Cambridge, CB2 1PZ, UK

Abstract

Metallic foams and lattice materials are compared in terms of properties and underlying deformation mechanisms. Lattice materials can be made to possess mechanical properties which are similar to, or exceed, those of foams depending upon the nodal connectivity. Scaling laws are reviewed here for the Young's modulus and yield strength of foams and lattice materials, and the damage tolerance of each class of material is discussed.

Introduction

Low density cellular structures appear widely in nature and are manufactured on a large scale by man. The cytoskeleton of living cells [1,2] the structures of cork, of wood , of trabecular bone, of coral [3] and of many other growing organisms [4] are described as "cellular" – a word coined by Hooke [5] to describe the structure of cork. Synthetic polymer foams, now a mature industry, are widely used in cushioning, packaging and energy-absorption applications [6]. More recently, a variety of polymeric, metallic and ceramic foams have been developed with the aim of manufacturing lightweight structures – notably sandwich structures – that are both stiff and strong [3, 7, 8]. In all these examples the mechanical properties are of central importance.

Cellular materials can be thought of as 'composites' made up of solid and space. The solid forms a set of connected struts or plates connected at their ends or edges, to give an open structure through which fluids can flow, or one in which the cells have faces and form closed spaces. Cellular materials are characterised by the solid of which they are made, by their relative density $\bar{\rho}$ (the fraction of space occupied by the solid), by the cell size and wall thickness, and by the connectivity and regularity of the cell walls and edges. We refer to the last two as 'cell-topology', meaning the aspects of the structure that depend on shape but not on scale. This paper concerns cell topology and its influence on mechanical properties.

Numerous studies on open-cell foams have shown that both their stiffness and strength are governed by cell wall bending for all loading conditions; the stiffness scales as $\bar{\rho}^2$ and the strength as $\bar{\rho}^{3/2}$ [3]. Most closed-cell foams also follow these scaling laws, at first sight an unexpected result because the cell faces must carry membrane stresses, and this should lead to a linear dependence of both stiffness and strength on $\bar{\rho}$. The explanation lies in the fact that the cell faces are very thin; they buckle or rupture at stresses so low that their contribution to stiffness and strength is small, leaving the cell edges to carry most of the load. Bending is a 'soft' mode of deformation; if the cell edges carried significant tensile loads both would be larger. So it is interesting to examine why bending dominates the deformation of almost all common foam-like structures, and whether other topologies exist in which bending is replaced by stretching.

Metallic foams are a promising class of engineering material, provided they can be manufactured at sufficiently low cost. Numerous techniques have been developed for synthesizing inexpensive metal foams with stochastic, closed or open cells. Some utilize open cell polymer templates for investment casting, chemical vapor deposition or slurry coating. Others utilize hollow spheres or aggregates of soluble particles into which metals can be injected and solidified. A leaching process removes the template leaving behind a cellular solid. While potentially less expensive to fabricate than benchmark honeycomb systems, their mechanical performance is inferior, because the metal ligaments experience bending when

Cellular Metals and Polymers 2004, edited by R. F. Singer, C. Körner, V. Altstädt
Fragezeichenverlag, Fürth, Long ISBN number 8585858585

loaded. Moreover, most have an as-cast ligament microstructure that contributes to low ductility.

The inferior structural performance of stochastic systems has spurred interest in periodic cellular metals that not only compete with honeycombs, but also offer multifunctionality [9-11]. The benefits arise when lattice materials are configured as cores within sandwich structures that experience either bending or compression. The cores of interest can comprise either prismatic elements (honeycomb, textile and corrugations) or an assembly of struts (tetrahedral, pyramidal, Kagome) or shell elements (egg-box). The truss and textile topologies can have either solid or hollow ligaments [12].

Rigidity of frames

To establish the topological criteria for stretch-dominated behaviour, we proceed to analyse the rigidity (or otherwise) of an assembly of inextensional pin-jointed struts.

Maxwell's criterion

Maxwell [13] suggested an algebraic rule setting out the condition for a pin-jointed frame of b struts and j frictionless joints to be both statically and kinematically determinate ie. to just be rigid. In 2 and 3 dimensions the criteria are

$$b = 2j - 3 \tag{1a}$$

and

$$b = 3j - 6 \tag{1b}$$

respectively. These criteria are necessary conditions for rigidity, but are not in general sufficient conditions as they do not account for the possibility of states of self stress (a self-equilibrated state of strut tensions in the absence of external load) and of mechanisms. A generalisation of the Maxwell rule in 3D is given by Calladine [14]:

$$b - 3j + 6 = s - m \tag{2}$$

where s and m count the states of self stress and mechanisms, respectively, and each can be determined by finding the rank of the equilibrium matrix that describes the frame in a full structural analysis [15]. A just-rigid framework (ie. a framework that is both statically and kinematically determinate) has $s = m = 0$. The nature of Maxwell's rule as a necessary rather than sufficient condition is made clear by examination of (2): vanishing of the LHS in (2) only implies that the number of mechanisms and states of self-stress are equal, not that each equals zero.

The role of nodal connectivity

The most common manufacturing route for foams is by bubble formation within the melt, and this leads to an open or closed cell microstructure such that the nodal connectivity is on the order of 3 to 4: the number of cell edges meeting at any node is between 3 and 4. Consider, for example the 3D tetrahedral diamond structure. It has a nodal connectivity of 4. It may be shown by structural analysis of pin-jointed structures that a high nodal connectivity, in excess of 3 for 2D structures, and in excess of 5 for 3D structures is needed for structural rigidity [11]. When structures possess a smaller connectivity than the critical value, the structure collapses as a mechanism if it is pin-jointed and behaves as a compliant, 'bending-dominated' structure if it has rigid joints: foams are of this type. In contrast, the octet truss structure, as shown in Fig. 1, is an example of a 3D lattice material with a sufficient nodal connectivity of 12 for it to behave as a redundant and stiff, 'stretching-dominated' structure regardless of whether the joints are pin-jointed or rigid. This simple notion has major implications for the engineering properties of foams and lattice materials.

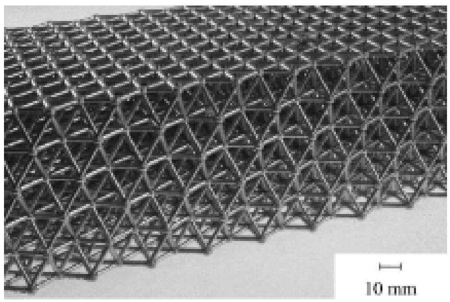


Figure 1: The octet-truss structure

The 2D structures shown in Fig. 2 emphasise these differences in response. The 4-bar structure of Fig. 2a behaves as a mechanism in its pin-jointed form, and deforms by bending of the beams in the rigid jointed version. The triangulated structure shown in Fig. 2b is statically and kinematically determinate in the pin-jointed form, and the rigid-jointed version is a stiff, stretching dominated structure. Such statically and kinematically determinate structures possess excellent morphing capability – upon replacement of a bar by an axial actuator, the structure can undergo large changes in shape without internal resistance. The structure shown in Fig. 2c is a redundant lattice with more bars per joint than required for rigidity. The redundancy of such structures can endow them with a degree of damage tolerance – if one bar fails the remaining structure may remain stiff. Damage tolerance is not guaranteed, and these structures also suffer from the drawback that they have internal states of self stress, which can be generated during fabrication – for example in the form of thermal residual stress. The structure of Fig. 2d has the correct number of bars to give it the possibility of being statically and kinematically determinate. However, the placement of the bars is such that the structure contains a single state of self stress and also contains a collapse mechanism.

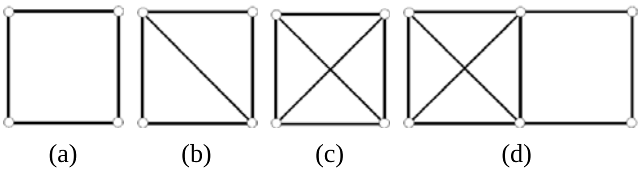


Figure 2: Finite, planar structures

Stiffness and strength of lattice materials

Simple beam theory can be used to determine the macroscopic Young's modulus E and Poisson ratio ν of lattice materials in terms of the relative density $\bar{\rho}$. The scaling law can be adequately represented by the power-law expression,

$$\frac{E}{E_s} = B\bar{\rho}^b \qquad (3)$$

where b=1 for a stretching dominated structure, such as those sketched in Fig. 3. In contrast, b=2 for a bending dominated structure, such as metallic foams. The pre-exponent B is dependent upon geometry but typically is on the order of 0.3 to 0.5.

Beam theory can also be used to determine the macroscopic yield strength of the perfect lattice, in the absence of heterogeneities such as a macroscopic crack. The strength σ_Y can be expressed in terms of the yield strength σ_{YS} of the cell wall materials according to

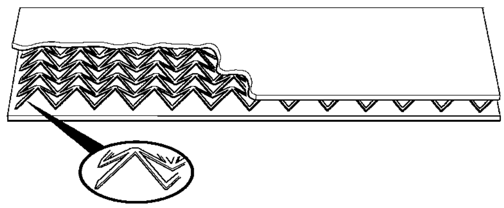
$$\frac{\sigma_Y}{\sigma_{YS}} = C\bar{\rho}^c \qquad (4)$$

with c=1 for a stretching dominated structure, such as those sketched in Fig. 3. In contrast, c=3/2 for a bending dominated structure, such as metallic foams. The pre-exponent C is dependent upon geometry but typically is on the order of 0.3 to 0.5.

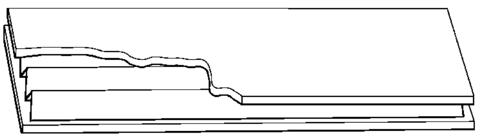
Representative compressive stress versus strain responses of steel and lattice materials and Alporas aluminium alloy foam are given in Fig. 4, all for a relative density of $\bar{\rho}$=5%. The steel designations, with measured strengths and energy absorption per unit volume W up to a nominal compressive strain of 0.55 are listed in Table 1. It is clear from Fig. 4 and from Table 1 that the lattice materials have a higher stiffness and a higher initial yield strength than the foam. The square honeycomb and pyramidal core also have a much higher energy absorbing capacity. Note that the fabricated lattice materials have a high compressive ductility – this is not the case for a cast structure such as the cast octet truss structure made from a high Si aluminium alloy [9] shown in Fig. 1.

Table I: Yield strength and energy absorption of competing steel lattice materials and Aluminium alloy foam, at a relative density $\bar{\rho}$ =5%

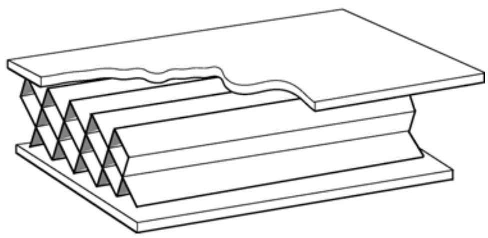
Material	Composition of cell walls	σ_{YS} MPa	σ_Y MPa	W MJm^{-3}	W/σ_{YS} mJ N^{-1}m^{-1}
Square honeycomb	304 stainless steel	210	10.9	3.13	14.9
Diamond core	304 stainless steel	210	3.33	0.74	3.5
Corrugated core	304 stainless steel	210	4.23	0.59	2.8
Egg box	Mild steel	200	1.21	0.34	1.7
Pyramidal core	Al6XN stainless steel	380	8.19	3.09	8.1
Alporas foam	Al-Mg alloy	200	0.63	0.47	2.3



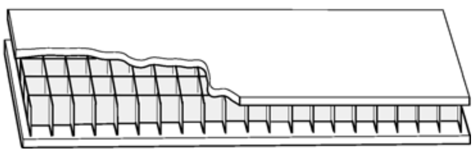
Pyramidal core



Corrugated core



Diamond core



Square Honeycomb core

Figure 3: Some lattice materials used as the core of sandwich plates

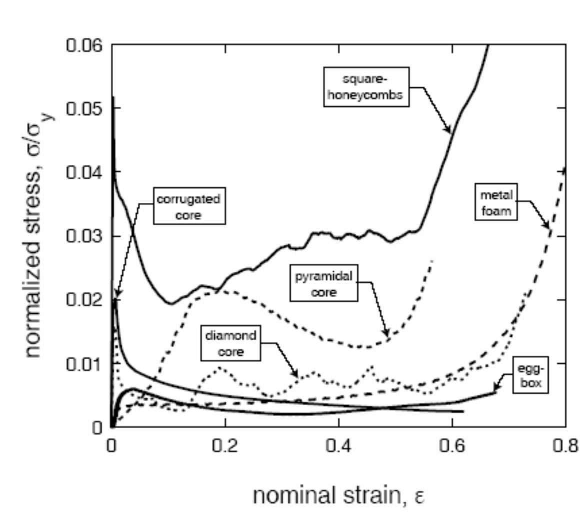


Figure 4: Compressive stress versus strain response of steel and lattice materials and Alporas aluminium alloy foam, all for a relative density of $\bar{\rho}$ *=5%*

Acknowledgements

The authors are grateful to ONR for their financial support through US-ONR IFO grant number N00014-03-1-0283 on The Science and Design of Blast Resistant Sandwich Structures.

References

[1] D. Stamenovic, N. Wang, Engineering approaches to cytoskeletal mechanics, J. Appl. Physics, 89, p. 2085 (2000)

[2] R. D. Kamm, (2002), Mechanics of the Cytoskeleton, to be published.

[3] L. J. Gibson, M. F. Ashby. Cellular solids: Structure and Properties, 2nd edition, Cambridge University Press, Cambridge UK (1997)

[4] J. F. V. Vincent. Structural Biomechanics, 2nd edition, Princeton University Press, Princeton NJ, USA (1990)

[5] R. Hooke, Micrographia, The Royal Society, London, p. 112 (1664)

[6] N. C. Hilyard, (ed.) Mechanics of Cellular Plastics, Applied Science Publishers, Barking, UK (1982)

[7] M. F. Ashby, A. G. Evans, N. A. Fleck, L. J. Gibson, J. W. Hutchinson, H. N. G. Wadley, Metal Foams: A Design Guide. Butterworth Heinemann, Oxford UK. (2000)

[8] J. Banhart, M. F. Ashby, N. A. Fleck. (eds), Proc. Int. Conf. Metfoam '99, (MIT Verlag, Bremen, 1999)

[9] V. S. Deshpande, M. F. Ashby, N. A. Fleck, Foam topology: bending versus stretching dominated architectures, Acta Materialia, 49, pp. 1035-1040 (2001)

[10] V. S. Deshpande, N. A. Fleck, M. F. Ashby, Effective properties of the octet-truss lattice material, J. Mech. Phys. Solids, 49(8), pp. 1747-1769 (2001)

[11] V. S. Deshpande, N. A. Fleck, Collapse of truss-core sandwich beams in 3-point bending, Int. J. Solids and Structures, 38, pp. 6275-6305 (2001)

[12] H. N. G. Wadley, N. A. Fleck, A. G. Evans, Fabrication and structural performance of periodic metal sandwich structures, Composites Science and Technology, 63, pp. 2331-2343 (2003)

[13] J. C. Maxwell, On the calculation of the equilibrium and stiffness of frames (1864)

[14] C. R. Calladine, Theory of Shell Structures. Cambridge University Press, Cambridge, U.K. (1983)

[15] S. Pellegrino, C. R. Calladine, Matrix analysis of statically and kinematically indeterminate frameworks. International Journal of Solids and Structures, 22(4), p. 409 (1986)