Spherical indentation behaviour of bitumen

E.A. Ossa, V.S. Deshpande, D. Cebon *

Cambridge University Engineering Department, Trumpington Street, Cambridge CB2 1PZ, UK

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Abstract

The spherical indentation response of a 50 pen bitumen is investigated both experimentally and via an analytical model. The model for the indentation of a power-law creeping solid of Bower et al. [Proc R Soc London, Ser A 1993;441:97–124] is extended to account for the recovery behaviour of bitumen by employing the concepts of effective stress and effective strain under the indenter. An extensive experimental study of the monotonic, recovery and cyclic spherical indentation behaviour is reported for a range of temperatures. In line with the predictions of the model, the monotonic indentation response of the bitumen exhibits a power-law dependence on the indentation force, while the continuous cyclic response is primarily a function of the mean indentation load. The model is also successful in capturing the indentation recovery behaviour of the bitumen and is shown to be in reasonable agreement with periodic pulse tests over a wide range of test conditions.

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1. Introduction

Asphalt is used in large quantities, world-wide, for road surfaces and airport runways. For example, in Europe, nearly 300 million tonnes of asphalt mix is produced annually. Asphalt roads suffer a variety of wear and failure mechanisms in service. For heavy duty roads, such as motorways, an important mode of failure is the formation of ruts due to permanent deformation and flow of the road surface material in the traffic wheel paths – particularly in low speed “truck lanes” [2].

Although there is a long history of empirical research into the deformation behaviour of asphalt in the civil engineering literature, there is surprisingly little understanding of the fundamental properties of the binder and the high volume fraction of angular aggregates which are combined to form an asphalt mix. The research described in this paper is part of an ongoing effort to develop micromechanical constitutive models of asphalt for use in pavement analysis and design.

Bitumen, which is commonly used as the binder for the aggregate in asphalts, is the highly viscous residue of crude oil, obtained by removing most of its volatile components. Early work by van der Poel [3] indicated that the “stiffness” of bitumen at low strains could be correlated with the penetration index and softening point of the bitumen [2]. van der Poel [3] summarised this behaviour in the well-known van der Poel nomograph [4]. Following the work of van der Poel, many researchers attempted to characterise bitumen as a linear viscoelastic material, using rheological models based on springs and dashpots and continuous relaxation spectra, see Ward [5] for a discussion on the viscoelastic models for polymers.

Under the US Strategic Highway Research Program [6] extensive studies were conducted in order to extend the linear viscoelastic models to more realistic conditions related to the specific application of bitumen, see for example [7–9]. These models required the construction
of master curves for the dynamic complex modulus and phase angle. In the construction of these master curves, the time temperature superposition principle, or reduced variables method was employed. However, these approaches have two major drawbacks. First, they are applicable only for relatively small strains (up to 0.1) and stresses in the linear viscoelastic range of behaviour, while the thin films of bitumen binder between the aggregate particles are subject to large stresses and strains. Second, a large number of experiments need to be conducted to calibrate these models.

Recently, Ossa et al. [10] proposed a phenomenological model for pure bitumen that captured its response over a wide range of temperatures, stresses and loading conditions, such as monotonic, continuous cyclic and intermittent cyclic loading. The model is simple and requires just four tensile tests to calibrate all the model parameters. The ability of the constitutive law of Ossa et al. [10] to capture the indentation response of bitumen is the focus of this study.

Indentation tests provide a cheap and easy method to measure the mechanical properties of materials and also serve to validate multi-axial constitutive models. The focus of this study is to investigate the monotonic and cyclic spherical indentation response of bitumen with the aim of (i) validating the multi-axial constitutive model for bitumen developed by Ossa et al. [10] and (ii) investigating the repeated indentation response of bitumen which serves as a unit problem for road surfaces under vehicle loads.

The standard indentation test on creeping solids involves either applying a constant load and measuring the indentation creep with time or by pressing the indentor into the material at a prescribed rate and measuring the load as a function of time. To interpret these results many researchers have developed models to relate the indentation pressure to the constitutive response of the materials. Notably, Tabor [11] proposed empirical relations to correlate the indentation pressure for rate independent strain hardening solids to the uniaxial tensile response of the material, while Mulhearn and Tabor [12] extended these empirical relations to power-law creeping materials. Using the similarity transformations for the indentation of metals developed by Hill et al. [13], Bower et al. [1] provided a rigorous theoretical basis for the empirical relations developed by Mulhearn and Tabor [12] for rate dependent solids. A source of error in the interpretation of creep properties from indentation tests is the neglect of the primary creep response (or the strain hardening behaviour) of rate dependent materials in the above analyses. Ogbonna et al. [14] extended the scaling procedure of Hill et al. [13] and Bower et al. [1] to a class of creep constitutive laws that account for strain hardening. Such analyses provides the basis for the investigation of the indentation response of bitumen reported in this study.

The outline of the paper is as follows. First the constitutive model for bitumen of Ossa et al. [10] is briefly reviewed. Second, the indentation model for power-law creeping solids of Bower et al. [1] is summarised and then extended to the constitutive model of Ossa et al. [10]. Finally, an extensive experimental study of the monotonic, recovery and cyclic spherical indentation behaviour of bitumen is reported for a range of temperatures, and the predictions of the model compared with experimental measurements.

2. Uniaxial deformation behaviour of bitumen

Cheung and Cebon [15] developed a deformation mechanism map for the steady-state deformation of bitumen over a wide range of temperatures, stresses and strain rates. For temperatures in the vicinity of the glass transition temperature $T_g$ of the bitumen and above, they showed that the transition from a linear viscous response at low stresses to non-linear viscous behaviour at high stresses is captured by the modified Cross model [16,15]. This model relates the steady-state stress $\sigma_{ss}$ and the steady-state strain rate $\dot{\epsilon}_{ss}$ through the relation

$$\frac{\sigma_{ss}}{\dot{\epsilon}_{ss}} = \frac{\sigma_o}{\dot{\epsilon}_p} \left( \frac{1}{1 + \left( \frac{\dot{\epsilon}_p}{\dot{\epsilon}_{ss}} \right)^n} \right),$$

where $\sigma_o$ and $\dot{\epsilon}_p$ are a reference stress and strain rate, respectively, and the power-law parameter $m$ governs the behaviour of the bitumen in the non-linear regime. The temperature dependence followed an Arrhenius type relation with

$$\dot{\epsilon}_p = \dot{\epsilon}_{pc} \exp \left\{ -k \left( \frac{1}{T} - \frac{1}{273} \right) \right\},$$

where $\dot{\epsilon}_{pc}$ is the reference strain rate at $T = 0$ °C or 273 K and $k$ the Arrhenius constant. At higher temperatures, the bitumen was found to follow WLF type behaviour like many polymers. Note that for high strain rates, $\dot{\epsilon}_{ss} \gg \dot{\epsilon}_p$, which corresponds to most practical operating conditions, the modified Cross model (1) reduces to a power-law creep relationship

$$\frac{\dot{\epsilon}_{ss}}{\dot{\epsilon}_p} = \left( \frac{\sigma_{ss}}{\sigma_o} \right)^n,$$

where $n = 1/(1 - m)$.

Ossa et al. [10] performed an extensive experimental investigation of the deformation behaviour of two pure bitumens (50 pen and 100 pen) over a wide range of temperatures, strain rates and stresses under monotonic, continuous cyclic and cyclic pulse loadings. Based on these experimental observations, they wrote the total strain rate $\dot{\epsilon}$ as the sum of a viscous strain rate $\dot{\epsilon}_v$, which
is active during loading (i.e. when the applied stress \( \sigma \neq 0 \)) and the recovery strain rate \( \dot{\varepsilon}^r \), which is only active when the stress \( \sigma = 0 \). Thus, for an arbitrary loading history, the total strain rate is given by

\[
\dot{\varepsilon} = \dot{\varepsilon}^p + \dot{\varepsilon}^r.
\]

(4)

In the high loading rate limit, the viscous strain rate is written in terms of the power-law relation

\[
\dot{\varepsilon}^p = \frac{\dot{\varepsilon}^p}{1 - \psi} = \dot{\varepsilon}_0(\varepsilon) \left( \frac{\sigma}{\sigma_0} \right)^n,
\]

(5)

where

\[
\dot{\varepsilon}_0(\varepsilon) = \dot{\varepsilon}_0 \exp \left\{ -k \left( \frac{1}{7} - \frac{1}{273} \right) \right\}
\]

(6)

and \( \dot{\varepsilon}^p \) is the irrecoverable fraction of the viscous strain rate while \( \psi \) is the “recovery constant” for the particular bitumen. The recovery rate is specified as

\[
\dot{\varepsilon}^r = -|\text{sign}(\varepsilon)|[1 - |\text{sign}(|\sigma|)|] \dot{\varepsilon}_0(\dot{\varepsilon}^r),
\]

(7)

where \( |\text{sign}(0)\rangle = 0 \) is defined to be zero,

\[
\dot{\varepsilon}^r = \left( \frac{\varepsilon}{\dot{\varepsilon}^p} - 1 \right) \frac{1 - \psi}{\psi}
\]

(8)

and

\[
\dot{\varepsilon}_0(\dot{\varepsilon}^r) = \dot{\varepsilon}_0 \exp \left\{ -k \left( \frac{1}{7} - \frac{1}{273} \right) \right\}.
\]

(9)

Note that \( \dot{\varepsilon}_0(\varepsilon) \) and \( \dot{\varepsilon}_0(\dot{\varepsilon}^r) \) are the loading and recovery calibration functions. These are unique functions of the strains \( \varepsilon \) and \( \dot{\varepsilon}^r \) and govern the strain hardening/softening behaviour of bitumen. Eqs. (4)–(9) can be integrated with respect to time to obtain the strain resulting from any applied stress history.

The constitutive model detailed above was developed for uniaxial loading. This model is generalised to multi-axial loading by noting that for all practical purposes, the response of the bitumen is independent of the mean or hydrostatic stress with the bitumen behaving like a rate-dependent von-Mises solid [15]. Adopting Cartesian tensor notation, the three-dimensional constitutive model can be written as

\[
\dot{\varepsilon}_{ij} = \dot{\varepsilon}^p_{ij} + \dot{\varepsilon}^r_{ij},
\]

(10)

A multi-axial generalisation of the uniaxial power-law model (5) based on the von-Mises effective stress is given as

\[
\dot{\varepsilon}_{ij} = \frac{\dot{\varepsilon}^p_{ij}}{1 - \psi} = \frac{3 \dot{\varepsilon}_0(\varepsilon)}{2} \frac{\sigma_0}{\sigma_0} \frac{\sigma_{ij}^{\text{dev}}}{\sigma_0}^{n-1} \frac{\sigma_{ij}^{\text{dev}}}{\sigma_0},
\]

(11)

where \( \sigma_{ij}^{\text{dev}} = \left( \frac{1}{2} \left[ \sigma_{ij}^{\text{dev}} \right]^{1/2} \right) \) is the von-Mises effective stress and \( \varepsilon_{ij} = \left( \frac{1}{2} \left[ \sigma_{ij}^{\text{dev}} \right]^{1/2} \right) \) the von-Mises effective strain with the primes denoting deviatoric quantities. Note that in a uniaxial test with the axial stress and strain \( \sigma \) and \( \varepsilon \), respectively, \( \sigma_0 = |\sigma| \) and \( \varepsilon_0 = |\varepsilon| \) for an incompressible solid. Consistent with the uniaxial observations, strain recovery is assumed to occur when \( \sigma_0 = 0 \) and the recovery constitutive relation (7) is generalised as

\[
\dot{\varepsilon}_{ij} = -[1 - \text{sign}(\varepsilon)] \dot{\varepsilon}_0(\dot{\varepsilon}^r_{ij}) \frac{\varepsilon_{ij}}{\varepsilon_0},
\]

(12)

where

\[
\dot{\varepsilon}_{ij} = \left( \frac{\varepsilon_{ij}}{\varepsilon_0} - 1 \right) \frac{1 - \psi}{\psi},
\]

(13)

The term \( \dot{\varepsilon}_{ij}/\varepsilon_0 \) ensures that (12) reduces to the uniaxial model and that volume constancy \( (\dot{\varepsilon}_{kk} = 0) \) is maintained.

3. Indentation behaviour of creeping solids

Consider a half-space, occupying the region \( x_3 \geq 0 \) and loaded by a frictionless spherical rigid indenter of diameter \( D \), as sketched in Fig. 1. The material in the half-space is assumed to deform according to a power-law creep law of the form

\[
\frac{\dot{\varepsilon}_{ij}}{\dot{\varepsilon}_0} = \frac{3}{2} \left( \frac{\sigma_0}{\sigma_0} \right)^{n-1} \frac{\sigma_{ij}^{\text{dev}}}{\sigma_0},
\]

(14)

where \( \sigma_0 \), \( \dot{\varepsilon}_0 \) and \( n \) are material constants.

Bower et al. [1] solved the problem of the plane strain and axisymmetric indentation of a half-space comprising a power-law creeping solid (14), using the similarity transformations suggested by Hill et al. [13]. These transformations are based on the observation that at any given instant, the velocity, strain rate and stress fields in the half-space only depend on the size of the contact \( a \) and the indentation rate \( h \), and are independent of the loading history. Thus, the general indentation problem is reduced to calculating stresses and displacements in a non-linear elastic solid, indented to a unit depth by a rigid flat punch of unit radius (in the

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Fig. 1. Spherical indentation of a half-space. The notation and sign convention is shown.
axisymmetric problem). For indentation by a frictionless spherical indenter, the similarity solutions dictate that the contact radius \( a \) is related to the indentation depth \( h \) by

\[
h = \frac{1}{c^2} \frac{a^2}{D},
\]

(15)

where the constant \( c \) is only a function of the material constant \( n \) and may be thought of as the ratio of the true to nominal contact radius, where the nominal contact radius is \( \sqrt{hD} \). Similarly, the applied load \( F \) is related to the indentation rate \( \dot{h} \) via

\[
\frac{F}{\pi a^2 \sigma_0} = \frac{\dot{h}}{c \epsilon_0} \left( \frac{h}{\epsilon_0} \right)^{1/n} = \frac{2a}{c^2} \left( \frac{2a}{c^2} h \right)^{1/n},
\]

(16)

where the constant \( \gamma \) is again only a function of the power-law exponent \( n \). Values of \( c \) and \( \gamma \) for selected values of \( n \) were deduced by Bower et al. \([1]\) from a series of finite element calculations and are listed in Table 1.

Eqs. (15) and (16) can be written in terms of an effective stress and effective strain under the indenter. The effective stress \( \sigma_{\text{eff}} \) under the indenter is defined as

\[
\sigma_{\text{eff}} = \frac{F}{\pi a^2},
\]

(17)

while the effective strain rate and strain under the indenter are specified as

\[
\dot{\varepsilon}_{\text{eff}} = \frac{\dot{h}}{D} = \frac{c\dot{h}}{2\sqrt{hD}}
\]

(18a)

and

\[
\varepsilon_{\text{eff}} = \sqrt{\frac{h}{D}},
\]

(18b)

respectively. Substituting these definitions in (15) and (16) gives the empirical results of Mulhearn and Tabor \([12]\).

\[
\sigma_{\text{eff}} = \gamma \sigma_0 \left( \frac{2\sigma_{\text{eff}}}{c^2 \epsilon_0} \right)^{1/n},
\]

(19)

for the pressure under an indenter in a power-law creeping solid.

### 4. Indentation model for bitumen

A model for both the monotonic and cyclic spherical indentation response of bitumen is proposed in this section. The model is based on the concepts of effective stress \( \sigma_{\text{eff}} \) (17) and effective strain \( \varepsilon_{\text{eff}} \) (18a) under a spherical indenter, introduced by Mulhearn and Tabor \([12]\) and justified theoretically by Bower et al. \([1]\).

As discussed in Section 2, the response of bitumen to load can be characterised by the generalised power-law relation (11) that accounts for the “strain hardening” or the primary creep response of bitumen. Ogbonna et al. \([14]\) extended the similarity relations of Bower et al. \([1]\), developed for the “steady-state” indentation of a power-law creeping solid to the indentation of a rate dependent strain hardening solid characterised by the Derby and Ashby \([17]\) constitutive relation. The finite element calculations of Ogbonna et al. \([14]\) showed that the steady-state analysis of Bower et al. \([1]\) provides an upper limit for the load factor \( \gamma \) and the ratio of the actual to nominal contact radius \( c \) with strain hardening not substantially affecting the values of these factors. We thus choose to employ the simpler Bower et al. \([1]\) analysis for the indentation of bitumen and extend it to account for the recovery behaviour.

The indentation rate \( \dot{h} \) is written as the sum of the viscous indentation rate \( \dot{h}^v \), which is active during loading (indentation force \( F \neq 0 \)) and the recovery rate \( \dot{h}^r \), which is only active when \( F = 0 \). For an arbitrary loading history, we write the strain rate \( \dot{\varepsilon} \) under the indenter as

\[
\dot{\varepsilon} = \dot{\varepsilon} + \dot{\varepsilon}^v = \frac{c}{2\sqrt{hD}} \dot{h} = \frac{c}{2\sqrt{hD}} \left( \dot{h}^v + \dot{h}^r \right),
\]

(20)

where the viscous indentation and strain rates are related to the indentation force via

\[
\dot{h}^v = \frac{2\sqrt{hD}}{c} \left( \frac{F}{\pi a^2 \sigma_0} \right)^{n} \epsilon_a(\dot{\varepsilon}),
\]

(21)

and \( \epsilon_a(\dot{\varepsilon}) \) in (16) has been replaced by the strain dependent function \( \epsilon_a(\dot{\varepsilon}) \). It now remains to specify the recovery indentation rate. We assume that the Mulhearn and Tabor \([12]\) definition of strain rate (18a) under the indenter is still applicable under unloading conditions, with recovery occurring in a self-similar manner; that is the ratio of the actual to nominal contact radius remaining constant at \( c \) as specified by the Bower et al. \([1]\) model. Thus, the indentation recovery rate is given by

\[
\dot{h}^r = \frac{2\sqrt{hD}}{c} \dot{h} = \frac{2\sqrt{hD}}{c} \left[ 1 - \text{sign}(|F|) \right] \epsilon_a(\dot{\varepsilon}),
\]

(22)
where $\dot{\varepsilon}^e$ is specified by (8) with the irrecoverable strain rate $\dot{\varepsilon}^p$ related to the viscous strain rate via the recovery constant $\psi$,

$$\dot{\varepsilon}^p = (1 - \psi)\dot{\varepsilon}^v.$$  \hspace{1cm} (23)

Eqs. (20) to (22) completely specify the monotonic and cyclic spherical indentation behaviour of bitumen: time integration of these equations provides the complete history of the indentation depth as a function of time for any specified loading.

5. Experimental investigation

5.1. Material

A 50 penetration grade (pen) bitumen was tested in this study. This bitumen is commonly used in hot rolled asphalt paving mixtures as well as in coated Macadam paving mixtures in the UK [2] and has been extensively characterised by Ossa et al. [10] in their uniaxial tensile deformation study. The material parameters for this 50 pen bitumen from the study of Ossa et al. [10] are listed in Table 2, and the loading calibration curve $\dot{\varepsilon}_oc(\varepsilon)$ and recovery calibration curve $\dot{\varepsilon}_uc(\varepsilon')$ are plotted in Figs. 2(a) and 2(b), respectively.

5.2. Specimen preparation

About 100 g of bitumen granules were taken from the freezer and melted at 160 °C for approximately 2 hours to remove all the air bubbles. The bitumen was then poured into the pre-heated cylindrical mould 1 60 mm in diameter and 50 mm in height. It was then allowed to cool to room temperature.

5.3. Test protocol

Spherical indentation tests on the specimens were performed in a hydraulic testing machine. The indentation load $F$ was measured using a 1 kN load cell, while the load line displacement was employed to obtain the indentation depth $h$. The spherical indenter of diameter 15 or 40 mm was lubricated with a thin layer of a mixture of soap and glycerine in order to prevent the specimen from adhering to it, allowing nearly frictionless indentation. Typically indents to a depth $h \leq 2$ mm were performed with the indent affected zone much smaller than the cylindrical mould dimensions. Thus, for all practical purposes, the tests may be regarded as indentation of a half-space of bitumen. The test temperature was controlled by performing the tests in an environmental chamber fitted on the test machine. The environmental chamber has a resolution of ±0.5 °C and the rates of loading employed in this study were slow enough for adiabatic heating effects to be negligible. Prior to testing, all specimens were kept in the environmental chamber for about 2 h to allow them to attain the test temperature. Unless otherwise specified, a 40 mm diameter spherical was employed in this study. A few tests were conducted with a 15 mm diameter indenter in order

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Table 2
Material parameters for the 50 pen bitumen from the uniaxial tensile study of Ossa et al. [10]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Uniaxial tension results</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi$</td>
<td>0.7</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>$22.8 \times 10^3$ K</td>
</tr>
<tr>
<td>$n$</td>
<td>2.6</td>
</tr>
<tr>
<td>$\sigma_0$</td>
<td>0.2 MPa</td>
</tr>
</tbody>
</table>

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Footnote:

1 The mould was pre-heated to 90 °C for 15 min, to avoid thermal contraction that could generate bubbles or residual stresses in the final specimen.

Fig. 2. (a) Loading and (b) recovery calibration curves from the uniaxial tensile experiments of Ossa et al. [10].
to confirm the predicted dependence of the indentation response on the indenter diameter. It is worth mentioning here that a number of spot repeat tests confirmed the reproducibility of the test results reported here. For the sake of brevity, these results are not presented.

5.3.1. Monotonic indentation tests

Constant indentation-rate and constant load creep indentation tests were employed to characterise the monotonic indentation response of the bitumen. In the constant indentation-rate tests, a specified indentation-rate \( \dot{h} \) was applied by the indenter and the resulting load \( F \) and indentation depth \( h \) recorded. In the constant load creep indentation tests, a constant indentation load \( F \) was applied “instantaneously” by the indenter and the indentation depth \( h \) recorded as a function of time \( t \).

5.3.2. Creep recovery indentation tests

The creep recovery indentation behaviour of bitumen was investigated by performing a series of single load/unload indentation tests as sketched in Fig. 3(a). A load \( F \) was applied rapidly by the indenter to the specimen and then held constant. The material was allowed to creep to a specified total indentation depth \( h^T \). At this indentation depth, the load was released and the indentation depth monitored until the indentation rate \( \dot{h} = 0 \). The indentation depth at this point \( h^{pl} = h^T - h^r \) is the irrecoverable indentation depth (Fig. 3(a)). Such tests were repeated for a series of indentation depths \( h^T \), loads \( F \), and temperatures.

5.3.3. Cyclic indentation tests

Continuous cyclic and pulse train indentation tests were performed to characterise the cyclic or repeated indentation behaviour of bitumen.

5.3.4. Continuous cyclic indentation tests

In the continuous cyclic indentation tests, the indentation load \( F \) was varied between \( F_{\text{min}} \) and \( F_{\text{max}} \) as sketched in Fig. 3(b), with

\[
R = \frac{F_{\text{min}}}{F_{\text{max}}}, \quad (24)
\]

and

\[
F_m = \frac{F_{\text{min}} + F_{\text{max}}}{2}, \quad (25)
\]

defining the load levels while the frequency \( f \) of the triangular waveform defines the loading rate. The indentation depth was measured as a function of time and tests repeated for a series of values of \( R \), \( F_m \) and \( f \).

5.3.5. Pulse train indentation tests

Tests comprising intermittent identical indentation load pulses with a trapezoidal shape in the time domain, as shown in Fig. 3(c), were performed in order to simulate a load history similar to that experienced in a pavement. The aim here was to investigate the relation between the single load/unload behaviour analysed via the creep and creep recovery indentation tests and the gradual ratcheting of indentation depth due to the
application of a continuous train of discrete load pulses as shown schematically in Fig. 3(c). The constant maximum indentation load \( F_p \) in each trapezoidal load pulse was applied for a time period \( \Delta t/2 \) with a loading and unloading rate \( \dot{F} = 4F_p/\Delta t \). A series of tests was performed by varying the time period \( \Delta t \) between consecutive trapezoidal pulses at a fixed \( F_p \) at two selected temperatures.

6. Experimental results and comparison with model predictions

The monotonic, recovery, continuous cyclic and pulse indentation experimental results are described in this section and the results compared with the predictions of the indentation model are described in Section 4. The material parameters employed in modelling the indentation behaviour are taken from Ossa et al. [10] and listed here in Table 2 with the loading and recovery calibration curves plotted in Fig. 2. Unless otherwise specified, the experiments reported were conducted at 0 °C employing the 40 mm diameter spherical indenter.

6.1. Monotonic indentation behaviour

The measured indentation load \( F \) versus indentation depth \( h \) response of the bitumen for two selected values of the applied indentation rate \( \dot{h} = 0.07 \) and 0.15 mm s\(^{-1}\) are plotted in Fig. 4(a). In both cases, the indentation load \( F \) increases monotonically with indentation depth and the slope of the \( F \) versus \( h \) curve increases with increasing indentation rate \( \dot{h} \).

Results from two constant load creep indentation tests \( (F = 40 \) and \( 100 \) N) are plotted in Fig. 4(b). The indentation depth \( h \) versus time response comprises two regimes: a primary creep regime where \( h \) decreases with time, followed by a secondary or steady-state creep regime, where \( h \) remains approximately constant. Increasing the indentation load \( F \) increases \( \dot{h} \) over the full range of indentation depths considered here.

The monotonic indentation responses at 5 °C employing the same loading parameters as in Fig. 4 are shown in Fig. 5. A comparison of Figs. 4 and 5 shows that while the qualitative behaviour remains unchanged, increasing the temperatures results in an smaller indentation force for a given indentation rate and a higher indentation creep rate for a given indentation force. Predictions of the model are included in Figs. 4 and 5 and show reasonable agreement with the experimental measurements at small times or indentation depths. However, the model generally predicts a “stiffer” response, that is, a lower indentation rate for a given indentation force or a higher force for a given applied indentation rate. This error arises from the approximation of the modified Cross model (1) by a power-law relationship: the power-law relationship over-predicts the creep resistance of bitumen, especially at stresses \( \sigma < \sigma_0 \).

The effect of the indenter diameter on the indentation response is illustrated in Fig. 6, where the indentation creep response at 0 °C for an indentation force \( F = 35 \) N is plotted for the two diameters of the spherical indenter \( D = 40 \) mm and \( D = 15 \) mm. In line with the predictions of the model the indentation creep rate \( \dot{h} \) increases with decreasing \( D \) with reasonable agreement between the model and the experimental measurements.

6.2. Creep recovery indentation behaviour

Creep recovery indentation tests on bitumen were performed at 0 and 5 °C and the recovery behaviour investigated for unloading from total effective strains \( e^T = c \sqrt{h^T/D} \) in the range 0.05 ≤ \( e^T \) ≤ 0.14. Experiments were performed for indentation loads of \( F = 65 \),
100 and 170 N for the 40 mm diameter indenter, and $F = 30$ N for the 15 mm diameter indenter. The creep recovery indentation response of bitumen at 0°C with $F = 65$ N is shown in Fig. 7(a), where the indentation depth $h$ is plotted as a function of time $t$, for the two selected values of $h^T = 0.7$ and 0.4 mm. The recovered indentation $h^r$ is higher for the larger $h^T$. This result is consistent with the observations of Ossa et al. [10] for bitumen in uniaxial tension, where it was shown that a larger initial tensile strain results in a larger recovered strain. A comparison between the model predictions and experimental measurements is included in Fig. 7(a) and indicates that the model captures both the loading and unloading indentation response of the bitumen reasonably accurately.

Results from all the creep recovery indentation tests performed are summarised in Fig. 7(b), where the recovered effective strain $\epsilon^r = c \sqrt{h^r/D}$ is plotted as a function of the total effective strain $\epsilon^T$ prior to unloading. The figure reveals that, to within experimental error, the data are well represented by the line $\epsilon^r = \psi \epsilon^T$ with $\psi (0 \leq \psi \leq 1)$ independent of the applied indentation load (effective stress), temperature and indenter diameter. Moreover, $\psi = 0.7$ as measured from these indentation experiments is approximately equal to that found by Ossa et al. [10] from uniaxial tensile experiments on the same bitumen. This was a key assumption made in the model detailed in Section 4 and is confirmed here through a wide range of experiments.

6.3. Continuous cyclic indentation response

The cyclic indentation depth $h$ versus time response of bitumen with $R = 0.7$ is shown in Fig. 8, for $F_m = 100$ and 150 N at 0°C. The cyclic load-controlled indentation response is similar in form to the monotonic creep indentation response with primary and secondary regimes of behaviour. Next, consider the influence of the load ratio $R$ and frequency $f$ on the cyclic load-controlled response. The indentation depth versus time history of bitumen with $F_m = 100$ N and $f = 5.0$ Hz is shown in Fig. 9(a) for three selected values of $R$ and in Fig. 9(b) with $F_m = 150$ N and $R = 0.7$ for three selected frequencies $f$. Both these figures demonstrate that the load ratio $R$ and frequency $f$ have a negligible effect on the cyclic load-controlled indentation depth versus time response of bitumen. Thus, similar to the findings of Ossa et al. [10] for bitumen subjected to uniaxial tension, the continuous cyclic indentation response is also primarily a function of the mean load $F_m$. 

Fig. 6. Indentation depth versus time history for a constant applied indentation force $F = 35$ N at 0°C. Experimental measurements and model predictions for two indenter diameters $D = 15$ and 40 mm are included in the figure.
Predictions of the indentation model are included in Figs. 8 and 9. Again, the model agrees reasonably well with the experimental measurements and correctly predicts the negligible dependence of the response on the load ratio \( R \) and frequency \( f \). Consistent with the monotonic response, the model predicts a stiffer response due to the approximation of the modified Cross relation (1) by a power-law model.

### 6.4. Pulse train indentation experiments

The indentation response at 0 and 5 \(^\circ\)C is plotted in Figs. 10(a) and (b), respectively, for cyclic load controlled pulse indentation tests with \( F_p = 75 \) N and \( \Delta_p = 4.0 \) s for two selected values of the pulse gap \( \Delta_g \) (see Fig. 3(c) for definitions of \( F_p, \Delta_p \) and \( \Delta_g \)). The results clearly show that for a fixed value of \( F_p \), the accumulated permanent indentation depth decreases with increasing \( \Delta_g \), because larger fractions of the creep strain are recovered in the zero-load gaps between the pulses. In fact, as \( \Delta_g \rightarrow 0 \), the pulse train tests converge to the continuous cyclic loading indentation tests, with no recovery of the accumulated strain.

A key measure of the accuracy of the model lies in its ability to predict the accumulated strain response of the bitumen in the pulse loading indentation tests. In these tests both the creep response of the bitumen under loads and its recovery behaviour is combined in a complicated manner and the response is integrated over many cycles resulting in the build-up of modelling errors. Such comparisons for pulse loading indentation tests are included in Fig. 10. The model underpredicts the total accumulated strain at large times similar to that observed in the monotonic and continuous cyclic loading cases. However, the errors in accumulated strain are only of the order of 10% and hence this simple model is considered adequate for practical purposes.

### 7. Discussion

The spherical indentation response of bitumen was investigated in this study and an extension to the model of Bower et al. [1] proposed to analyse the monotonic and cyclic indentation response of bitumen. The analysis of Bower et al. [1] is based on the observation that the fields under the indenter in a power-law creeping solid are self-similar, which reduces the analysis to calculating stresses and displacements in a nonlinear elastic solid, indented to a unit depth by a rigid flat punch. In fact, Bower et al. [1] presented a general
analysis which is applicable to either plane strain or axisymmetric indenters of arbitrary geometries including conical and cylindrical indenters. The extension to the Bower et al. [1] model presented here could also be generalised to these cases on lines similar to that presented in Section 4.

The self-similar analysis of Bower et al. [1] is strictly valid for effective strains under the indenter $\varepsilon_{\text{eff}} \leq 0.2$: beyond these strain levels, finite strain effects play a significant role and the simple model presented here is expected to be unable to capture the indentation response. Full finite element solutions of the field equations would be necessary to obtain the indentation response in such cases.

In the present study, the frictionless indentation limit was investigated by coating the indenters with a mixture of soap and glycerine. A few spot tests were conducted with non-lubricated indenters in order to gauge the effect of adhesion on the indentation behaviour. These tests revealed that the indentation recovery behaviour is substantially affected by the adhesion of the bitumen to the indenter. Further investigation into the effects of adhesion between the indenter and bitumen is proposed as a topic of future study.

8. Concluding remarks

The similarity solution, developed by Bower et al. [1] for the indentation of a power-law creeping solid, has been extended to the constitutive model of Ossa et al. [10] for the monotonic and cyclic response of bitumen. Under non-zero applied loads, bitumen behaves as a power-law creeping solid and the analysis of Bower et al. [1] is applicable. During unloading, the bitumen is assumed to recover in a self-similar manner and the
effective strain rate under the indenter continues to be related to the indentation depth by (18a) and (18b).

Employing this strain measure in the constitutive relation for unloading bitumen it is possible to characterise the indentation behaviour of bitumen under both monotonic and cyclic loading conditions.

Monotonic, continuous cyclic and cyclic pulse loading experiments were conducted over a range of temperatures. Similar to the uniaxial tensile behaviour of bitumen, the continuous cyclic response was observed to depend mainly on the mean applied indentation load while the cyclic pulse loading behaviour depended strongly on the recovery behaviour of bitumen and hence was affected by the rest periods in the loading history. The proposed indentation model captures the experimentally observed indentation response accurately over a wide range of loading conditions. Moreover, the model is also successful in predicting the temperature dependence of the indentation response and the effect of the indenter diameter.

The monotonic and repeated indentation behaviour investigated here is the unit problem for understanding the behaviour of pavements under vehicle loads and is thus of intrinsic interest. Moreover, the indentation study has helped validate the multi-axial constitutive model for bitumen developed by Ossa et al. [10]. Extending this work to understand the repeated indentation behaviour of asphalt (bitumen plus aggregate) will be a topic of future investigations.

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References